**Table 1. Arrangements of Nancarrow’s primarily canonic studies**

<table>
<thead>
<tr>
<th>Study No.</th>
<th>Ratios</th>
<th>Arrangement(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 14</td>
<td>4:5</td>
<td>Helena Bugallo, Raaf Hekkema, Ivar Mikhashoff, Dorrance Stalvey</td>
</tr>
<tr>
<td>No. 15</td>
<td>3:4</td>
<td>Hekkema, Mikhashoff (multiple)</td>
</tr>
<tr>
<td>No. 16</td>
<td>3:5</td>
<td>Hekkema, Erik Oña, James Tenney</td>
</tr>
<tr>
<td>No. 17</td>
<td>12:15:20</td>
<td>Mikhashoff</td>
</tr>
<tr>
<td>No. 18</td>
<td>3:4</td>
<td>Hekkema, Mikhashoff, Oña</td>
</tr>
<tr>
<td>No. 19</td>
<td>12:15:20</td>
<td>Bugallo, Mikhashoff</td>
</tr>
<tr>
<td>No. 21</td>
<td>“X”</td>
<td>Dominic Murcott (with two conductors using click tracks)</td>
</tr>
<tr>
<td>No. 22</td>
<td>1:1/3:2/3:4/3:6/3</td>
<td>none</td>
</tr>
<tr>
<td>No. 24</td>
<td>14:15:16</td>
<td>none</td>
</tr>
<tr>
<td>No. 26</td>
<td>1:1</td>
<td>Bugallo, Wolfgang Heisig, Hekkema, Nancarrow, Matt Rogers, Stalvey</td>
</tr>
<tr>
<td>No. 27</td>
<td>5% : 6% : 8% : 11%</td>
<td>none</td>
</tr>
<tr>
<td>No. 31</td>
<td>21:24:25</td>
<td>Paul Usher</td>
</tr>
<tr>
<td>No. 32</td>
<td>5:6:7:8</td>
<td>Bugallo, Stalvey</td>
</tr>
<tr>
<td>No. 33</td>
<td>2:√2</td>
<td>Usher</td>
</tr>
<tr>
<td>No. 34</td>
<td>9:10:11</td>
<td>Nancarrow</td>
</tr>
<tr>
<td>No. 36</td>
<td>17:18:19:20</td>
<td>none</td>
</tr>
<tr>
<td>No. 40</td>
<td>e : π</td>
<td>none</td>
</tr>
<tr>
<td>No. 41a</td>
<td>1/√3 : √2/3</td>
<td>none</td>
</tr>
<tr>
<td>No. 41b</td>
<td>1/3 : √13/16</td>
<td>none</td>
</tr>
<tr>
<td>No. 43</td>
<td>24:25</td>
<td>none</td>
</tr>
<tr>
<td>No. 44</td>
<td>Indeterminate tempo ratio</td>
<td>Bugalo</td>
</tr>
<tr>
<td>No. 47</td>
<td>5:7</td>
<td>none</td>
</tr>
<tr>
<td>No. 48</td>
<td>60:61</td>
<td>none</td>
</tr>
<tr>
<td>No. 49</td>
<td>4:5:6</td>
<td>Thomas Adès</td>
</tr>
</tbody>
</table>
Figure 1. 12:15:20 tempo ratio as a polyrhythm in a single notated tempo

\[ \frac{J}{4} = 360 \left( \frac{J}{4} = 120, \frac{J}{4} = 72 \right) \]

Figure 2. Opening of Nancarrow's Study No. 33
Figure 3. Nearly aligned chords in the opening of Nancarrow's Study 33
Figure 4. Opening of Usher's arrangement of Study 33

\[ \text{\textit{j} = ca.64} \]

\textbf{Violin I}

\textbf{Viola}

\[ \frac{17}{12} \]

\[ \approx \sqrt{2} \]

\[ \approx 2\sqrt{2} \]

\textbf{Violin II}

\textbf{Violoncello}
Figure 5. Nearly aligned chords in Usher's arrangement
Figure 6. Usher's arrangement of Study No. 33, Canon 3
Figure 7. Alternate notation for violin 2 in Study No. 33, Canon 3, measure 55

\[ \begin{array}{c}
\text{\rotatebox{90}{\scriptsize 11}} \\
\text{\rotatebox{90}{\scriptsize 3}} \\
\text{\rotatebox{90}{\scriptsize 11}} \\
\text{\rotatebox{90}{\scriptsize 3}}
\end{array} \]

Figure 8. 7:5 tempo ratio with a convergence period of \( \frac{7}{8} \) seconds

\[ \varphi \approx 69 \quad \frac{7}{8} \text{ sec.} \]

Figure 9. Convergence periods for (a) 10:7 tempo ratio, (b) 17:12 tempo ratio, and (c) 24:17 tempo ratio

\[ \varphi = 48 \quad \frac{10}{8} \text{ sec.} \]

\[ \varphi \approx 28 \quad \frac{17}{8} \text{ sec.} \]
Figure 10. 5+6+6 division in the tempo ratio $\frac{24}{17}$

$= 60$

Figure 11. 11+6 division in the tempo ratio $\frac{24}{17}$
Figure 12. Hypothetical version of canon 3, approximating tempo ratio

![Sheet Music](image1)

Figure 13. Hypothetical version of canon 3, approximating beat placement

![Sheet Music](image2)
Figure 14. Nancarrow's approximation of $\sqrt{2}$, Study No. 33, systems 56–60
The first few durations are marked as multiples of a sixteenth note.
Figure 15. (a) Converging-diverging canon, (b) Diverging-converging canon, and (c) Abstract canonic structure of Study 33

These three structural diagrams are adapted from Gann 1995

Figure 16. Tempo exchange, canon 1, Nancarrow
Figure 17. Realization of the tempo exchange in canon 1 that yields an unnecessarily complex rhythmic notation after the exchange

Figure 18. A different realization of the same tempo exchange that yields a much simpler rhythmic notation
Figure 19. Tempo exchange, canon 1, Usher

Faster tempo

beats:

\[ 15\sqrt{2} \quad 16\sqrt{2} \quad 17\sqrt{2} \quad 18\sqrt{2} \quad 19\sqrt{2} \quad 19\sqrt{2} + 1 \]

Slower tempo

Figure 20. Tempo exchange, canon 3, Nancarrow

exchange gap

\[ \approx 18 \text{ ms} \]
Figure 21. Tempo exchange, canon 3, Usher

Faster tempo

Slower tempo

147 sixteenths

104\sqrt{2} sixteenths

Slower tempo

Faster tempo
Figure 22. Tempo exchange, canon 4, Nancarrow

\[ J = 140 \times \sqrt{2} \]

\[ (\frac{7}{4}) 898.5 \text{ eighths} \]

exchange gap
\[ \approx \frac{6}{5} \text{ eighth notes} \]

\[ J = 280 \]

\[ (\frac{7}{4}) 634.5 \sqrt{2} \text{ eighths} \]
Figure 23. Tempo exchange, canon 4, Usher
Figure 24. Opening of canon 2, Nancarrow

echo distance
\[ = 4\sqrt{2} + \frac{1}{2} \]
whole notes
Figure 25. Opening of canon 2, Usher

echo distance approximately 6 and 1/10 quarters
Figure 26. Opening of canon 4, Nancarrow

\[ t = 140 \times \sqrt{2} \]

\[
\begin{align*}
79\sqrt{2} & & 78\sqrt{2} & & 77\sqrt{2} \\
\end{align*}
\]

Echo distance = \(79\sqrt{2} - 79\)

\(\approx 32 \frac{2}{3}\) quarter notes or \(8\frac{1}{6}\) whole notes
Figure 27. Opening of canon 4, Usher

[Music notation image]
Figure 28. Opening of canon 3 with Arditti String Quartet recording

(canon 3)
Figure 29. Graph of beats with respect to time and the resulting trend lines in the recording of the opening of canon 3.
Figure 30. Inter-onset intervals in the original and recorded versions of the opening of canon 3

Figure 31. (a) First violin measure 68 as performed and (b) as notated