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Types and Applications of $P_{3,0}$ Seventh-Chord Transformations in Late Nineteenth-Century Music

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ABSTRACT: The expression $P_{3,0}$ refers to one class of parsimonious voice-leading transformations between seventh chords introduced in a 1998 article by Jack Douthett and Peter Steinbach as $P_{m,n}$ (*Journal of Music Theory* 42 (2): 241–63). In addition to tones that may be held in common, the subscripts indicate the number of voices that move by half step (m) or whole step (n) in connecting one seventh chord to the next. $P_{3,0}$ designates a transformation in which one of the chord members is held in common while each of the other three moves by half step. $P_{3,0}$ transformations produce some of the most striking chromatic harmonic progressions in the late Romantic repertoire. This study focuses on aspects of $P_{3,0}$ transformations that include 1) their place in the broader context of neo-Riemannian voice-leading transformations; 2) their properties and a specific means of notating all possible $P_{3,0}$ types; 3) explications of how the various types are integrated within late nineteenth-century harmonic practice and interact with traditional tonal harmony; and 4) analytic applications that demonstrate how $P_{3,0}$ transformations operate within and contribute to musical structure, including the opening of the Prelude to Wagner's *Tristan und Isolde*, and a complete song ("*Ruhe, meine Seele!*" op. 27 no. 1) by Richard Strauss.

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I. $P_{3,0}$ in the Context of Neo-Riemannian Voice-Leading Transformations

[1.1] $P_{3,0}$ refers to one class of parsimonious voice-leading connections between seventh chords introduced in an article by Jack Douthett and Peter Steinbach (1998) as $P_{m,n}$. In addition to common tones that may be held between the two chords, the subscripts indicate the number of voices that move by half step (m) or whole step (n) in progressing from one chord to another. Thus, the smaller the sum of m and n, the more parsimonious the connection, and the more closely related the two chords are from a neo-Riemannian perspective. $P_{3,0}$ indicates a relationship between two seventh chords in which holding one tone in common and moving each of the other three by half step will transform one chord into the other.

[1.2] Numerous other studies of voice-leading transformations in nineteenth-century chromatic harmony appeared around the time $P_{m,n}$ was identified and in the years since. A comprehensive survey of these investigations is not practical here, but it is useful to note that studies of transformations based on such common-tone-plus-stepwise voice leading tend to fall into three general categories, as listed below:

1. Triadic transformations, usually among major and minor triads, or set class 3-11 (037); see Cohn 1996, Cohn 1997, and Kopp 2002.(1)
2. Seventh-chord transformations among chord types of the same set class, especially dominant and half-diminished, or set class 4-27 (0258); see Lewin 1996, 205ff., Childs 1998, and Callender 1998.
3. "Cross-type" transformations among:
    a. triads *and* seventh chords; see Capuzzo 2004, Hunt 2007, and Hook 2007;(2) or
    b. triads *or* seventh chords of non-equivalent set class; see Douthett and Steinbach 1998 and Tymoczko 2011, 284ff.(3)

[1.3] These and related studies typically construct *Tonnetze* or similar networks to illustrate how specific kinds of voice-leading transformations create discernible, logical pathways through harmonic spaces.(4) Frequently, the networks presented have been limited to transformations among consonant triads, and only a small percentage of these studies have sought to explore through analysis how transformational voice-leading interacts with voice-leading associated with traditional harmonic practice. Steven Rings, for instance, has observed that a neo-Riemannian analysis often "[begins] with the most chromatically exceptional moment in the piece and [moves] outward from there to construct a broader interpretation" (2011, 492). In these cases, considerations of key and harmonic syntax are undervalued in favor of tonally neutral chord networks.(5)

[1.4] One of the advantages of revisiting $P_{m,n}$ is that it is inherently generic. That is, while it has potential for the construction of networks and associated graphs (as Douthett and Steinbach so effectively demonstrate), it also possesses the flexibility to operate independently of such networks, and in ways that facilitate the analysis of music that blends tonally unencumbered voice-leading transformations with tonal harmonic syntax.

[1.5] Regarding transformations in the second and third categories above, $P_{m,n}$ is particularly attractive for its applicability to a large number of seventh-chord connections involving parsimonious voice leading and chords of either the same or different set classes. To date, however, analytical studies that examine seventh-chord connections from this perspective have tended to focus on transformations with specific subscript values. Douthett and Steinbach, for example, explore only $P_{1,0}$ and $P_{0,1}$ among chords of different set classes (dominant/half-diminished sevenths and minor sevenths) and $P_{2,0}$ among chords of a single set class (dominant/half-diminished sevenths). The $P_{3,0}$ relationship has not been discussed in any detail as a specific transformation class, yet it holds considerable interest in its ability to account for a variety of seventh-chord successions in late nineteenth-century music. The role it plays within the host of common-tone and chromatic voice leading paradigms developed as a means of extending the range of harmonic relationships beyond the routines of the Classical style.

[1.6] Our article reconsiders this particular transformation from an analytical perspective. This entails an explanation of how $P_{3,0}$ relates to other transformation classes and to traditional harmonic practices, and an identification of specific $P_{3,0}$ types and their properties based on details of the voice-leading motions that connect one seventh chord to another. The larger part of our study is concerned with the ways in which $P_{3,0}$ transformations operate within harmonic structures from the late nineteenth-century repertoire.

II. Types and Notation of $P_{3,0}$ Relationships

[2.1] Although $P_{3,0}$ transformations do occur in some traditional harmonic progressions, they also occur in ambiguous or unstable situations that at least temporarily suspend tonal centricity. The harmonic characteristics of specific $P_{3,0}$ transformations depend on the chord types involved and the chromatic movement of the voices in relation to one another, as explained in the following discussion.

[2.2] The possible relationships between the antecedent (first) and consequent (second) harmonies produced by $P_{3,0}$ are numerous and diverse, and in every instance the transformation results in a change of chord quality.(6) In fact, all $P_{3,0}$ transformations belong to the "cross-type" category; the set class is always changed. They can also be divided into two groups according to the nature of the motion among the three moving voices: those in which the semitonal steps are in parallel motion, and those in which one voice moves contrary to the other two. The possible seventh-chord relationships belonging to the "parallel" $P_{3,0}$ group are given in **Table 1**, while the "contrary" transformations are shown in **Table 2**. The set of objects represented is limited to the most common seventh-chord types associated with nineteenth-century tonal practice: major ($^M7$), dominant (7), minor ($^m7$), half-diminished ($^ø7$), diminished (°7), augmented dominant (7♯5), and French sixth (7♭5).(7)

[2.3] Whereas none of the transformation types represented in Tables 1 and 2 connects seventh chords that are diatonically related, some do correspond with familiar chromatic harmonic relationships, such as the common-tone diminished seventh that embellishes a dominant seventh harmony (e.g., C°7–C7 in F major), and the normative resolution of a French sixth to a dominant seventh (e.g., C7♭5–B7 in E minor). Of particular interest are the relationships at the other end of the spectrum, such as dominant and minor seventh chords (e.g., C7–F♯^{m}7), and major and dominant seventh chords (e.g., C^{M}7–F♯7) that cannot be explained in terms of tonal harmonic syntax. In some instances, the juxtaposition of $P_{3,0}$-related seventh chords produces collections of pitch classes that are subsets of the octatonic collection (e.g., C7♭5–A7) or other modes of limited transposition. The nature of the $P_{3,0}$ transformation (both contrary and parallel) is such that, in every case, the sum class changes from antecedent to consequent harmony, both in magnitude and in parity (that is, a tetrachord with an even sum class is transformed into one with an odd sum class).(8) The sum classes of the members of the prime form set 4-27 (dominant and half-diminished sevenths) are odd, while the remaining tetrachords relevant to this study all have even sum classes. This explains in part the outputs seen in Tables 1 and 2; the dominant and half-diminished seventh chords are able to generate members of all the other seventh chord types examined here, while lacking the ability to move within their own set class.(9) That is, every $P_{3,0}$ transformation listed involves motion to or from a dominant or half-diminished seventh chord.(10)

[2.4] Almost all of the chord-quality relationships between the antecedent and consequent harmonies are the same regardless of whether the transformation belongs to the parallel or contrary group, but they differ in terms of the relationships between the chord roots. A notable exception is the diminished seventh chord, as none of the other chord types are associated with it as a direct result of contrary $P_{3,0}$ transformations. Contrary motion in this situation necessitates that the interval between two moving voices either expands or shrinks by a whole step, as those two voices shift by one semitone each. Because three of the four voices move by semitone in the $P_{3,0}$ transformation, there must be one pair of adjacent voices that undergoes a whole step expansion or shrinkage. The stacked intervals that make up a dominant seventh and half-diminished seventh chord are all either minor or major thirds (and an inverted major second between the root and chordal seventh). A whole-tone expansion or contraction of any of these intervals due to contrary semitone motion would result in at least one interval that is not a minor third. Since the diminished seventh chord is a set of symmetrically stacked minor thirds, it cannot be achieved by application of the contrary $P_{3,0}$ transformation.

[2.5] **Table 3** presents a model for the analytic notation of specific types of $P_{3,0}$ transformations. Three factors determine the form and function of these transformations: 1) the relative interaction and direction of the voice leading, in either parallel or contrary motion; 2) the chord factor of the common tone between the antecedent and consequent harmonies; and 3) the chord factor of the independent chord tone opposing the other moving voices in a contrary $P_{3,0}$. A parallel $P_{3,0}$ depends only upon factors 1 and 2, while a contrary $P_{3,0}$ is controlled by all three. Table 3 presents two related methods of notation for the distinct $P_{3,0}$ types.

[2.6] Contrapuntal motion from the antecedent to the consequent chord is indicated by a lowercase "p" in parallel motion and "c" in contrary motion. Immediately following that is an arrow identifying the direction of the three moving voices for parallel types or the majority direction for contrary types. The remainder of the parallel $P_{3,0}$ notation is a letter or number in parentheses indicating the chord factor (root, third, fifth, or seventh) of the common tone, based on its position in the antecedent chord.(11) The contrary $P_{3,0}$ has an additional notation in square brackets preceding the common-tone factor, indicating the chord factor of the voice that moves in opposition to the other two voices (and thus opposite to the direction of the arrow). This table lists the specific analytical notation for each transformation type described in Tables 1 and 2.(12) For example, the parallel motion $P_{3,0}$ that relates C7 to B♭^{m}7 is $P_{3,0}$ p↑(7), indicating that the transformation retains the seventh of the C7 chord (B♭) as a common tone, while the remaining tones move upward by half step (C–D♭, E–F, G–A♭). Similarly, the contrary motion $P_{3,0}$ that connects C^{m}7 to F♯7 is $P_{3,0}$ c↑[5](7), which retains the seventh of the C^{m}7 (B♭/A♯) as a common tone while its root and third move up by half step and the fifth moves independently downward (C–C♯, E♭–E, G–F♯). The fact that individual $P_{3,0}$ types may result in several distinct chordal relationships is an important consideration, as a chain of $P_{3,0}$ functions can act as a compositional procedure that connects seventh chords of varying qualities (as discussed in the analysis of Richard Strauss's op. 27, no. 1 in Part IV below).

[2.7] A remarkable property of the $P_{3,0}$ transformation is that its voice leading is the complement of that for $P_{1,0}$ and, in the abstract, it has a close relationship to Douthett and Steinbach's "OctaTowers" and "Power Towers" graphs.(13) The respective qualities of the antecedent and consequent seventh chords achieved by $P_{3,0}$ and $P_{1,0}$ are the same, although they differ in the relationship between the chord roots and, of course, pc content. Similarly, the octatonic relationships implied by $P_{1,0}$ are also relevant for $P_{3,0}$, such that many of the seventh-chord combinations form octatonic subsets. Of particular interest are the contrary $P_{3,0}$ transformations, because they form clear octatonic networks and connections between these networks when applied systematically to tertian tetrachordal sets. Thus the Power Towers and OctaTowers of Douthett and Steinbach can be constructed in a similar form for contrary $P_{3,0}$ transformations, with different chord roots. Illustrations of these structures are provided in Examples 1 and 2. Analogous to the OctaTowers is **Example 1**, the "OctaCrystal" formed by the contrary $P_{3,0}$ transformation when applied to (0258), (0268), and (0358) prime-form tetrachords within the Oct0,1 set. Among these tetrachord types, the OctaCrystal is a closed network;(14) however, if major sevenths and augmented dominant sevenths are incorporated, "bridges" between octatonic sets are formed, and a complete "Power Crystals" network can be constructed, demonstrating the connections among all three octatonic sets. A partial version of this Power Crystals graph is given in **Example 2**.(15) This network shows major seventh chords as bridges between the OctaCrystal $P_{3,0}$ networks with dashed lines.

[2.8] Another consequence of the similarity between $P_{3,0}$ and $P_{1,0}$ is the possibility of reversing chord-quality transformations by combining the two. For example, a $P_{3,0}$ that connects a dominant seventh to a minor seventh with a tritone-related root, or $P_{3,0}$ c↓[r](3), may be coupled with a $P_{1,0}$ moving the third of the minor seventh up by half step to yield a larger-scale connection between two dominant sevenths separated by a tritone , or $P_{3,0}+P_{1,0}\rightarrow P_{2,0}$. This type of compound transformation is employed in the Romantic repertoire as a compositional tool that allows for connections between distantly related seventh chords of the same quality (as in Franck's Piano Trio in D Major, op. 2, discussed in Part III below, and Wagner's Prelude to *Tristan und Isolde*, discussed in Part IV).(16)

III. Examples of $P_{3,0}$ in Nineteenth-Century Repertoire

[3.1] Applications of the $P_{3,0}$ transformation can be sorted into three basic groups. The first of these consists of parsimonious voice leading that coincides with a traditional harmonic paradigm; the second contains $P_{3,0}$ transformations constructed as a compound of additive semitonal motions; and the third is made up of novel, non-functional progressions, conceived entirely in terms of voice-leading parsimony and usually with no discernible relation to traditional harmonic syntax.

[3.2] The most common progressions in the first category are the embellishment of a dominant seventh harmony by a common-tone diminished seventh chord, and the conventional progression of a French sixth as a pre-dominant leading to a dominant seventh, as noted in Part II above. In both of these cases, the semitonal motion is parallel. The procedure for a common-tone diminished seventh is the voice-leading complement of a $P_{1,0}$ transformation from dominant seventh to diminished seventh a half step above (e.g. C7–C♯°7). **Table 4** compares $P_{1,0}$ and $P_{3,0}$ initiating from specific harmonies (C7 and C°7) as an example of this complementary voice leading. The symmetrical property of the diminished seventh chord enables it to reach four different dominant seventh chords via $P_{1,0}$ and also via $P_{3,0}$ when it is the antecedent. In tonal harmonic practice, the difference between $P_{1,0}$ and $P_{3,0}$ in this situation is that the former is associated with the common progression vii°7–V7 (in various inversions), whereas the latter is a common-tone diminished seventh chord embellishing a dominant seventh harmony. A similar $P_{3,0}$ relationship is employed in the parallel voice-leading motion from a French sixth to either of two dominant sevenths, at the half step or perfect fifth below, corresponding to the tonal progression Fr+6–V7 in two tritone-related tonalities.

[3.3] Figure 1 illustrates three characteristic uses of $P_{3,0}$ that fall into this first category. The opening of Richard Strauss's "Das Geheimnis" op. 17, no. 3 in A-flat major (**Figure 1a**) serves to establish the mood of the 1888 piece and the evasive nature of its topic. It consists of a common-tone diminished seventh chord that alternates with a third-inversion dominant seventh harmony. The $P_{3,0}$ p↓(r) transformation that introduces the C diminished seventh on beat two is then reversed to become $P_{3,0}$ p↑(3), as it returns to the E♭7.

[3.4] The passage from Franck's Piano Trio op. 2, shown in **Figure 1b**, obfuscates the B-minor tonality in measures 71–79 through the $P_{4,0}$ oscillation between two diminished seventh chords with B♯ and C♯ in the bass and an acceleration of harmonic rhythm leading to the last two beats of measure 80. At that point, the pattern is altered, with B♯°7 interpreted as a common-tone diminished seventh leading to F♯7 as a functional dominant that introduces the ensuing B-major theme. In essence, the role of this $P_{3,0}$ is to couple with the $P_{1,0}$ in measure 73, transforming B7 to B♯°7 to F♯7. This also accomplishes a $P_{1,0}$ transformation of the repeated C♯°7 to F♯7 as indicated in the figure. An element of tonal instability is thus converted into a mechanism for re-establishing the B tonality.

[3.5] **Figure 1c** (Scriabin's Prelude op. 11, no. 2, measures 7–12) features an abundance of stepwise voice leading, including motion from a French sixth to a dominant seventh in measures 8–9. Whereas the pitch classes of this $P_{3,0}$ transformation correspond to those of a traditional Fr+6 to V7 progression in D minor, there is nothing else in this passage to suggest that tonality, and even the ultimate key of the prelude, A minor, is not strongly represented at this point in the piece. Whereas the harmonic content is reminiscent of tonal practice, in this musical context the progression is more usefully understood as a chromatic voice-leading transformation.

[3.6] **Figure 2** illustrates the two most common forms of compound semitone motions that result in $P_{3,0}$. The excerpt from Richard Strauss's "Die Zeitlose" op. 10, no. 7 shown in **Figure 2a** demonstrates the $P_{2,0}+P_{1,0}\rightarrow P_{3,0}$ paradigm as a means of text painting. The first eight measures clearly express the tonality of G major as the text describes a bucolic pasture of saffron flowers; however, coinciding with the key signature change in measure 9 is a shift in the mood of the text, which discloses the flowers' poisonous nature. At this point, the voice leading in the piano part becomes exclusively stepwise and principally semitonal. As indicated in the harmonic reduction of **Figure 2b**, measures 8–9 contain a $P_{2,0}$ transformation from C7 to $C^{ø}7$ followed by the half-step descent ($P_{1,0}$) B♭ to A, converting the half-diminished chord to fully diminished. The relationship between the chord of measure 8 and the last beat of measure 9 is thus $P_{3,0}$. In measures 11–12 there is another compound $P_{3,0}$ that reverses the order of transformations of the first, as a $P_{1,0}$ creates a fully diminished seventh after which $P_{2,0}$ produces the $A^{m}7$ harmony, and the progression continues with stepwise voice leading. These $P_{3,0}$ relationships give the passage its distinctive character, and they are anticipated by the opening common-tone diminished seventh figure embellishing the G tonic, which places the same transformation in a different context.

[3.7] An excerpt from the beginning of Mily Balakirev's song "Kogda volnuyetsya zhelteyushchaya niv" (When the Yellow Cornfield Waves), shown in **Figure 2c**, embodies a similar harmonic procedure that departs from conventional tonal practice in favor of semitonal voice leading. The increase in energy indicated by the *Poco più animato* marking begins with a parallel $P_{3,0}$ common-tone diminished seventh in the piano in measure 27, followed by $P_{1,0}$ transformations on each of the next three beats that effect a reversal of the $P_{3,0}$ and return to the initial G7 harmony. The two intermediate vertical harmonies are $G^{ø}7$ on beat 4 of measure 27, and a French sixth (G7♭5) on the downbeat of measure 28, before the final half-step motion rearticulates G7. An identical progression (not shown) encompasses the following two measures of music, transposed to begin on C7. These $P_{3,0}$-related chords, connected via successive half-step shifts, serve to intensify the drama near the close of this song.

[3.8] The most harmonically striking types of $P_{3,0}$ transformations are those creating connections between harmonies related in ways that do not coincide with any progression associated with tonal practice, despite the fact that the individual chords involved are tertian constructions that have the potential to behave functionally. Progressions of this sort frustrate established expectations and force an interpretation that is apart from, even though it may also share space with, tonal writing. Both parallel and contrary $P_{3,0}$ transformations carry the potential for many such connections, including those between tonally unrelated dominant and minor seventh chords, major seventh and dominant seventh, or French sixth and dominant seventh, which are among the progressions considered below.

[3.9] A common Romantic-era implementation of parallel $P_{3,0}$ in novel harmonic progressions connects dominant and minor sevenths with root relationships that are either tonally distant or unrelated to the surrounding tonal context. The connection generated by the $P_{3,0}$ p↑(7), for example (namely C7–$B♭^{m}7$, as shown in Table 3), corresponds with the association between V7 and iv7 in F minor, but this potential functional relationship may not be realized. This is the case with the two examples given in Figure 3. Richard Strauss's 1885 Lied "Die Verschweigenen" op. 10, no. 6 (**Figure 3a**) employs a dominant-to-minor seventh parallel $P_{3,0}$ transformation that lacks a direct connection to the tonal structure of the passage. The piece opens by establishing A minor as the tonal center, before A7 is introduced in measure 7. Subsequently, two equal but opposite iterations of the parallel $P_{3,0}$ transformation convert the A7 to $G^{m}7$ and back. The progression may serve on a larger level to foreshadow the D minor key area that arrives later, in measure 16, but in the context of the passage shown, the relation between A7 and $G^{m}7$ is not functional. Essentially, the harmonic objective of the progression is to prolong A7 with chromatic neighboring motion, while simultaneously increasing the level of chromaticism in the transition as it leads to the new key area. The same pair of seventh chords appears at the start of a section of Rachmaninoff's song "Duma" op. 8, no. 3, which emphasizes parsimonious voice leading (measures 21–25). In this instance, represented by the harmonic reduction in **Figure 3b**, the progression can be entirely understood by transformational analysis up to the final dominant-tonic progression in C minor. It begins with the $P_{3,0}$ relationship between A7 and $G^{m}7$, which is then transposed to connect G7 to $F^{m}7$ and subsequently altered to create two $P_{3,0}$ transformations that set up the functional dominant.

[3.10] Another relationship involving dominant and minor seventh chords, but one that lacks potential tonal implications, is shown in Table 3 as $P_{3,0}$ p↑(3) connecting C7 and C♯^{m}7. In this relationship the notes forming the interval of a minor seventh in the dominant seventh chord move in parallel to become the same interval in the minor seventh chord. **Figure 3c** offers an example of this transformation in a harmonic reduction of a passage from the end of Liszt's symphonic poem *Ce qu'on entend sur la montagne*, in E-flat major. The section at rehearsal W is in the distantly related key of F♯ minor, which is emphasized in the upper voices, while at the same time obfuscated by the moving bass. B^{m}7 arrives as iv7 of F♯ minor and is prolonged until a parallel $P_{3,0}$ transforms it into B♭7, which then resolves as the dominant of the principal key at rehearsal X.

[3.11] Two less common forms of the parallel $P_{3,0}$ transformation are illustrated in Figure 4. The first of these is an excerpt from Hugo Wolf's "Genialisch Treiben." The final phrase of text ("es ist ein Nichts/und ist ein Was") presents a contradiction perhaps more metaphysical and complex than the other dualisms in the poem (namely "earnest/jest," "love/hate," "this/that"). In this passage, the musical setting (measures 39–47) utilizes three repetitions of the $P_{3,0}$ p↑(7) shift from major seventh to dominant seventh, as indicated in the harmonic reduction given as **Figure 4a**. Initially, these transformations are in sequence, until a deviation in measure 43. Subsequently, another $P_{3,0}$ relates B♭^{M}7 to B7, followed by a $P_{4,0}$ alternation with C7. This repetitive alternation is discontinued in measure 46, where a $P_{2,0}$ introduces F♯7, whose role is to tonicize B major. Structurally, the three sequential iterations of $P_{3,0}$ between measures 39 and 45 allow the bass to progress chromatically from A to E; the subsequent cadential figure has an essentially pre-dominant–dominant–tonic foundation, with the bass motion E–F♯–B.

[3.12] The fourth musical phrase from Liszt's "Ich liebe dich" is shown in **Figure 4b**. In this song, each line of text begins with the titular phrase, followed by a reason for, metaphor about, or description of the power of love. This fourth line of text, "ich liebe dich durch einen Zauberbann," is the only part of the poem that invokes the supernatural, as the poet's love is attributed to some magical power. Liszt's setting of these words initially incorporates a common-tone diminished seventh via $P_{3,0}$ p↓(r), from B7 to G♯°7. Subsequently, on the word "Zauberbann," a $P_{1,0}$ transforms F°7 into F♭7, which in turn becomes the antecedent for another $P_{3,0}$ from F♭7 to the consequent augmented dominant harmony, E♭7♯5. This sets up a cadence in the principal key of A♭ at measure 28. Throughout the transformations in this eight-measure phrase, B natural, which is the uppermost pitch of the vocal line, is retained as the only common tone, and assumes four different roles within the harmonic succession before ultimately resolving upward to C at the cadence.

[3.13] Contrary $P_{3,0}$ transformations produce chord progressions that are more diverse and often more harmonically progressive than their parallel counterparts. The two excerpts shown in Figure 5 exemplify contrary $P_{3,0}$ connections between dominant and minor seventh chord types, with different chord root relationships. The opening of the Balakirev song in **Figure 5a** features a connection between chords with tritone-related roots (C♯7 and G^{m}7) that evokes an atmosphere of ambiguity and a lack of tonal focus. This initial progression prefigures the tonal conflict between F major and D minor that occurs throughout the piece, juxtaposing C with C♯ and B with B♭ in stepwise, semitonal motion. The piece's major-minor dynamic is only convincingly abolished at the close, with a plagal cadence in F major and a subsequent four measures of obsessive, *fortissimo* F major triads. Additionally, parallel $P_{3,0}$ motion occurs in this piece in multiple iterations, a sign that parsimony is a relevant feature of the work as a whole (recall Figure 2c). **Figure 5b**, from Rachmaminov's song "Duma," a few bars after the passage shown in Figure 3b, illustrates another version of contrary $P_{3,0}$ that connects a dominant seventh to a minor seventh, but differs in terms of which chord factor among the three moving voices is contrary to the other two, and which chord factor is held as a common tone. It also reverses the order of the chord types, with the F^{m}7 as the antecedent, followed by D7. The resultant transformation in this case is between chords with roots that are related by a minor third rather than by a tritone.

[3.14] Two less common examples of contrary $P_{3,0}$ are given in **Figure 6**. The first of these involves the connection between a dominant seventh and augmented dominant seventh whose roots are related by a minor third. This excerpt is from the early portion of Strauss's opera *Elektra* op. 58, from 1908 (**Figure 6a**). Each of the chords (D♭7 and B♭7♯5) has the potential to resolve as a dominant-functioning harmony, but in this context they relate only to one another, connected by semitonal voice leading, and both harmonizing the A♭ in the highest voice. Another contrary $P_{3,0}$ progression occurs three measures later, connecting F♯^{m}7 to E♭7. **Figure 6b** shows a subtler contrary $P_{3,0}$ from the piano introduction to Liszt's "Der du von dem Himmel bist" in E major (first setting, 1842): namely, a French sixth moving to a functional dominant seventh. This progression, however, does not correspond to the resolution associated with traditional tonal practice. The entire introduction prolongs the dominant as a bass pedal. The harmony moves to a half-diminished supertonic at the beginning of measure 5, after which the French sixth unfolds across two octaves leading to V7, a $P_{3,0}$ c↑[7](3) (taking D as the root of the French sixth). The normal resolution of this French sixth to a dominant seventh would be to either C♯7 or G7 as the dominant of F♯ or C, but in this case it moves to B7 with F♯ (the third of the D7♭5) as the common tone. The motion from C to B, as scale degrees $\flat\hat{6}$ to $\hat{5}$ in E major, nevertheless infuses the progression with a palpable suggestion of subdominant-to-dominant function.

IV. Analytic Applications: The Opening of Wagner's Prelude to *Tristan und Isolde* and Strauss's "Ruhe, meine Seele!"

[4.1] The preceding discussion deals with examples from the repertoire that exhibit $P_{3,0}$ transformations of various types, and it focuses on the explication of mostly localized, chord-to-chord connections. This final section seeks to demonstrate some ways in which these transformations operate within and contribute to larger musical structures.

[4.2] The opening of the *Tristan* Prelude has been fertile ground for numerous efforts at explaining its tonal and harmonic content, including approaches based on harmonic function, sequential construction, linear-motivic structure, or even pitch-class set organization.(17) It has also been an attractive example for neo-Riemannian analysis based on parsimonious voice leading, especially as it exhibits Douthett and Steinbach's $P_{2,0}$-type transformations in the sequential passage consisting of the first eleven measures.(18) Further aspects of the harmonic content of this passage, however, can be elucidated by considering $P_{3,0}$ relationships as well.

[4.3] The opening harmonic sequence of the Prelude is given in **Figure 7a**, with a harmonic reduction showing the chords formed by the four voices sounding simultaneously in **Figure 7b**. The sequence is made up of a model followed by two copies in ascending-third transpositions. As David Lewin points out, $P_{2,0}$ relationships exist between the half-diminished ("Tristan") chord on the downbeat of measure 2 and the dominant seventh at the end of the model (indicated above the staff in Figure 7a), and also between the Tristan chord and each of the initial and final chords of both copies. The same relationship obtains between the initial and final chords of the first copy, but it does not continue in the altered second copy, where the half-diminished chord at the beginning and the dominant seventh at the end are $P_{4,0}$-related, having no pitch classes in common.(19) Parsimonious voice leading is clearly the driving force of the harmony in this passage, but $P_{2,0}$ does not offer an entirely consistent explanation for its construction.

[4.4] As indicated at measure 10 of Figure 7a, there is an added symmetrical triadic harmony (C augmented) in the second copy. The pitch class sets of the model and the first copy each consist of two chromatic tetrachords related at T6. The second copy, however, constitutes a complete octatonic collection plus one additional note—the E of the augmented triad that has no precursor earlier in the sequence. The deviations from the pattern that occur in measures 9–11 seem to indicate that there is motion through the sequence *toward* this second copy of the sequence, a perspective illustrated by the annotations beneath the reduction in Figure 7b that identify $P_{3,0}$-related chords resulting from the combination of successive semitonal movement. The embedded $P_{3,0}$ architecture is retained in the second copy, but the added augmented triad forces the transformation to shift from the parallel to the contrary variety. The implications of the overlapping $P_{3,0}$ analysis are multifaceted. First, because there is no precedence given to any particular chord within each three-measure statement, it is possible to account for both the half-diminished sevenths and the French sixths in the passage. Also, the preservation of the overlapping $P_{3,0}$ organization in measures 9–11 retains the prevalent common-tone and semitonal relationships from the previous statements in the otherwise altered second copy. As for the pitch-class sets employed in the three-measure statements, the shift to contrary $P_{3,0}$ in measures 9–11 is also responsible for the introduction of the octatonic collection.

[4.5] A characteristic of many contrary $P_{3,0}$ transformation types (and few of the parallel variety) is that the combined pitch-class content of the antecedent and consequent chords produces a nearly complete octatonic collection. One of the diminished seventh chords contained in the octatonic set of the second copy in measures 9–11 is B°7, which, as Cohn demonstrates, pervades the entire sequence (Cohn 2012, 150). The four pitch classes of that harmony are the only common tones retained in the model and both copies. Finally, as indicated by the arrows in the harmonic reduction, each of the two pairs of $P_{3,0}$-related seventh chords shares a single but different common tone; these notes, taken together, induce a voice exchange between the soprano and tenor voices. In the final, altered statement in measures 9–11 (with the contrary $P_{3,0}$ and inserted augmented triad) the voice exchange between the soprano and tenor differs from the others in that it has a slight rhythmic realignment of the notes that are exchanged. Many significant attributes of the model Tristan-chord progression are preserved in measures 9–11, in large part as a consequence of the overlapping $P_{3,0}$ transformations.

[4.6] It is possible to imagine several alternatives to measures 9–11 of the Prelude, but none would be as satisfactory as the original because it would entail either a failure to conclude on B7 as necessary preparation for the V7 of A minor that resolves deceptively to VI in measure 17, or a failure to retain the pitch classes of B°7 as common tones throughout the three iterations of the sequence, while still preserving the voice exchange that is an essential characteristic of the construction. The overlapping $P_{3,0}$ relationships result from the integration of semitonal voice leading and a voice exchange that provides the harmonic structure for the passage as a whole. Alterations to the progression in measures 9–11 create deviations within that structure in order to develop new seventh-chord relationships that maintain the same contrived attributes, in this case the pitch classes of B°7 as common tones.

[4.7] Whereas the opening of the Tristan Prelude contains $P_{3,0}$ relationships that are embedded within its series of seventh-chord sonorities and overlap with other relationships (especially $P_{2,0}$), Strauss's "Ruhe, meine Seele!" op. 27, no. 1 (1895) is remarkable for its employment of $P_{3,0}$ as an explicit procedure for generating chord-to-chord successions, as well as for the delineation of formal divisions in the piece. This song has been cited in earlier studies for its lack of conventional formal structures and its often ambiguous tonal orientation. Based on its text (given in **Figure 8**), it is possible to view the song as five melodic phrases ending with a semicolon, period, dash, or exclamation point (Hain;" "Sonnenschein." "schwillt." "Not—" "bedroht!").[20] The musical segments corresponding to these divisions, however, do not provide similarly strong tonal-harmonic punctuation. Marie Rolf and Elizabeth Marvin write, "If a phrase has beginning, middle and end, it is difficult to find one complete phrase in all of 'Ruhe, meine Seele!'" (Rolf and Marvin 1990, 72). In this work, parsimonious voice leading is the principal harmonic procedure, while traditional tonal syntax assumes an ancillary role.

[4.8] The opening four measures of "Ruhe, meine Seele!" act as a microcosm of the motivic and voice-leading procedures developed later in the work. This progression, shown in **Figure 9a**, is remarkable at the outset for the contrary $P_{3,0}$ transformation connecting C7 and F♯^{m}7. The same $P_{3,0}$ type recurs two more times at points of structural articulation: measures 22–23 (F7 to B^{m}7) and 39–40 (returning to C7 and F♯^{m}7). This harmonic figure is coupled with a stepwise descending bass motion that Rolf and Marvin refer to as the "Rest Motive," which recurs in various forms later in the piece, including measures 14–20, 22–30, and 31–34. Its final appearance is in the last five measures, where it descends chromatically above a C pedal that provides the tonal closure of the work.

[4.9] Another feature of the opening four measures has to do with its pitch-class content. The opening C7–F♯^{m}7 pair make up a seven-note subset of one octatonic collection ($\subset Oct_{0,1}$), and the chords in measures 3 and 4 each belong to one of the other two octatonic collections (D^{ø}7 $\subset Oct_{2,3}$ and C♯7 $\subset Oct_{1,2}$). The mingling of different octatonic subsets in this way avoids the implication of a specific tonality and suggests a linear harmonic conception emphasizing semitonal voice leading. The choice of these particular seventh chords is also remarkable in that they include every pitch class of the aggregate except D♯, which does not appear as a chord tone until measure 11, where the word "Sonnenschein" is articulated and the piano arpeggiates upward to D♯7, the highest pitch of the piece.

[4.10] A harmonic reduction and audio example of the complete song are given in **Figure 9b**; the score is provided in **Figure 10**. As the harmonic reduction indicates, parsimonious seventh-chord transformations are pervasive in this work (with $P_{3,0}$ most prominently represented), and progressions with tonal implications are sparse until the end, where C gradually emerges as the harmony associated with the repose that is the central idea of the text.[21] Momentum towards C begins to accumulate in measures 34–35, with an authentic resolution of A7 to D (ii). A cadential progression toward C minor ensues, but the resolution is thwarted when the expected tonic is replaced with C7 (V7/iv) at measure 39, and the opening (C7–F♯^{m}7–D^{ø}7) is restated. This time, however, the D^{ø}7 that has no discernible function in measure 3 acts as a subdominant added-sixth that resolves plagally to C major. Even the final harmonic closure of the piece, then, lacks a traditional authentic cadence, and is also inflected with the same tonally indeterminate harmonies as the beginning.

[4.11] In between the fleeting progressions associated with tonal procedures (e.g., the circle-of-fifths segment in measures 4–13 and some isolated chord pairs in measures 26–32) there are additional $P_{3,0}$ transformations of the parallel variety. One example occurs after measure 14, where a $P_{4,0}$ (B7–C7) obliterates the preceding circle of fifths and then moves to G^{m}7 ($P_{1,1}$) at measure 16. The ensuing progression employs two iterations of the $P_{3,0}$ p↓(r) type (G^{m}7–A7–F♯°7) followed by a $P_{1,0}$ that establishes F7 (measure 22), which in turn becomes the antecedent for the transposed version of $P_{3,0}$ c↓[r](3) from measures 1–2. A second example of a parallel $P_{3,0}$ of the same type ($P_{3,0}$ p↓(r)) is used to set up the A7 in measure 34 that begins the functional path toward the C tonic. The strategic placements of both the contrary and parallel varieties of the $P_{3,0}$ transformation in this song thus serve to clarify the musical structure, even though that structure is in many ways far removed from traditional tonal and formal practice.

[4.12] This study seeks to codify aspects of $P_{3,0}$ voice leading in a pragmatic way that encourages their incorporation into analyses focused on harmonic procedures associated with nineteenth-century chromaticism. Like most investigations based on neo-Riemannian transformations, the approach is limited in its ability to proffer all-encompassing explanations of complete tonal-harmonic constructions; rather, it serves as an effective resource for acquiring a deeper understanding of certain aspects of harmony in this repertoire not adequately addressed through traditional methods. The frequent appearance of $P_{3,0}$ transformations and relationships in works by a variety of composers, and especially those occurrences that represent extraordinary departures from established practice, indicates that there is value in considering the various roles $P_{3,0}$ may assume in local and larger-scale structures as it combines with other kinds of parsimonious voice-leading transformations and, especially, in the ways that all these procedures cohabitate with the routines of tonal harmony. The short analyses presented above represent some initial possibilities, and further study would in all likelihood uncover other ways in which such interactions take place, and how they cooperate in the creation of non-traditional types of musical coherence.

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Footnotes

1. Important precedents for the studies listed here are Lewin 1982 and Hyer 1989.
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2. This category is prefigured to an extent in Douthett and Steinbach 1998 and Santa 2003; the term "cross-type" comes from Hook 2002 and Hook 2007. For Hook, cross-type transformations involve a change in pc-set cardinality (e.g., a triad becoming a seventh chord). Here, a more generalized usage of "cross-type" refers primarily to transformations that change the set class, either by cardinality or by prime form.
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3. Of particular interest are the graphs and discussions of "OctaTowers" and "Power Towers" in Douthett and Steinbach 1998, 255–57. In Tymoczko 2011, single semitone motions produce cross-type transformations, which the author then interprets as accumulating towards transpositions of a dominant seventh chord.
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4. See, for example, Cohn 1998 and Cohn 2000.
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5. In the latter portion of his study of Schubert's Impromptu in G-flat Major (500ff.), Rings finds it necessary to abandon the distinctive properties of seventh chords by subsuming them into a network based solely on triadic content; he does this in an effort to create a context in which tonal-harmonic syntax and transformational networks can be considered side by side.
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6. This property of $P_{3,0}$ stands in marked contrast to $P_{2,0}$, in which dominant and half-diminished seventh chords (set class 4-27) are organized into cycles within octatonic space, as discussed in various sources (see category 2 above).
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7. It would be possible to add other chord types to the set of objects in the tables, but doing so would greatly increase the number of $P_{3,0}$ types with practically no relevance for analysis of the repertoire examined in this study. Such additions would include the mM7 (minor triad with major seventh), AM7, and the dM7. Three other combinations of triad plus seventh are transpositionally equivalent to other types already noted: md=dm (half-diminished seventh), Md=mm (minor 7),

and Ad=mM. A similar situation arises with the French sixth sonority: using the (026) trichord (e.g., C–E–G♭) as the triadic part of the seventh chord with minor seventh above (B♭), substituting a diminished seventh instead (A) is equivalent to a dm7, and substituting a M7 (B) produces a non-tertian (0157) sonority.
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8. Sum class is determined by adding together the pitch-class numbers of the chord. For example, the sum class of C7 is 0+4+7+10= 9 (mod 12). Set classes of a particular cardinality have either an even or odd sum class and, as explained below, this determines which set classes can be transformed into others via a specific $P_{m,n}$ class such as $P_{3,0}$.
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9. In Childs 1998, for example, $P_{2,0}$ transformations leave the sum-class parity unchanged and are used to form networks within the transpositions and inversions of set class 4-27.
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10. Another way of explaining this would be to consider the "parsimonious voice leading sum," or PVLS, as defined in Santa 2003 (a variation of the DVLS proposed in Cohn 1998), of each transformation. PVLS is essentially the mod-12 magnitude difference between sum classes of the antecedent and consequent chord in a transformation; for parallel $P_{3,0}$, PVLS = 3, while for contrary $P_{3,0}$, PVLS = 1. This means that the $P_{3,0}$ transformation imposes an odd number shift in sum class, necessitating a switch in the parity of the sum class from even to odd or vice versa (even + odd = odd; odd + odd = even).
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11. For the purposes of the present study, the chord root is generally determined by stacking the tetrachord as a tertian seventh chord in root position. For symmetrical pc sets, namely the diminished seventh and French sixth (7♭5), the root is either the note that is in the bass (and the chord factors relate to this note as root of the chord) or it is that which corresponds most closely to the tonal context of the music (the most probable root based on key and normative resolution). Context takes precedence over chord spelling in such cases, as enharmonic spellings become commonplace in the nineteenth-century repertoire.
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12. Employing a notation based in part on identifying chord factors enables the analyst to describe the motion of each individual voice within a given chord-to-chord motion. However, the dependency on tertian spelling obfuscates the inversional relationships between certain transformations. In particular, when the roots of the antecedent and consequent chords are different, the inverse of a given $P_{3,0}$ transformation is rarely a simple flip of the direction of the arrow in the notation described here. For instance, motion from C7 to B♭^{m}7 involves the $P_{3,0}$ p↑(7) transformation, while the inverse, B♭^{m}7 to C7, uses $P_{3,0}$ p↓(r). The arrow switches direction, but the common-tone chord factor is also changed (for other examples, see Figures 1a, 3a, and 6a). In cases where the root is the common tone, there may be instances where the inverse transformation only involves a switch of the arrow's direction, as in the progression G7–G°7–G7 (see Figure 2c). Generally, if the roots of the two harmonies are related by a larger interval than a semitone, the chord-factor notation is likely to differ between the original and inverse transformations.
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13. The term "voice-leading complement" refers to a relationship between two transformations such that the moving voices are mutually exclusive; for example, B^{ø}7→ B°7 (A moves to A♭) vs. B^{ø}7→ B♯°7 (B–D–F move to B♯–D♯–F♯).
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14. In the case of the contrary $P_{3,0}$ OctaCrystal, the PVLS (see note 10 above) of each branch is 1. The branches are independent, such that two successive iterations of a contrary $P_{3,0}$ can connect two $P_{2,0}$-related seventh chords, with a composite PVLS of 0 (i.e., C7–E♭7, passing through F♯^{m}7). This is a characteristic particular to the contrary version of the $P_{3,0}$ transformation.
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15. For clarity, some of the possible connections (including all cases where an augmented dominant seventh chord connects two OctaCrystals in a similar manner to the major sevenths shown) are omitted in the Power Crystals graph provided. Comprehensiveness in this situation would result in an unwieldy and unnecessarily complex graph.
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16. Among seventh-chord transformations including strictly semitonal voice leading, $P_{2,0}$ and $P_{3,0}$ have the largest number of possibilities, especially with regard to those involving contrary motion. There exists a parallel $P_{3,0}$ transformation that is the voice-leading complement for every $P_{1,0}$, as well as the full set of contrary $P_{3,0}$ transformations. Parallel $P_{4,0}$ is trivial in that it is limited to semitonal transposition of the same chord type, and only a limited number of possibilities exist for contrary $P_{4,0}$ (major→major, major→minor, minor→minor, dominant→half diminished, and, perhaps most interesting, French sixth→augmented dominant).
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17. That which is considered to be the "opening" may be limited to the first three measures, which contain the "Tristan chord" and its progression (perhaps via a French sixth) to a dominant seventh harmony, or it may extend as far as measure 17, where the same dominant seventh resolves (albeit deceptively) to the work's first consonant triad.
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18. See especially the discussions in Lewin 1996, 207–9; Cohn 2000, 101; and Cohn 2012, 149–57.
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19. For one interpretation of the altered aspects of this sequence, see Bass 1996, 279–84.
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20. Strauss's setting replaces the period after "schwillt" with another exclamation point in measure 25, and replaces a comma (perhaps an editorial error) with a period after "Ruhe" in measure 31.
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21. Whereas Figure 10 identifies various $P_{m,n}$ transformations, specific types are noted only for $P_{3,0}$ in order to elucidate its importance in the harmonic structure of the piece. The motivic stepwise bass descents are indicated by brackets above the lower staff in the reduction.
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