MTO 25.1 Examples: Willis, Comprehensibility and Ben Johnston’s String Quartet No. 9

(Note: audio, video, and other interactive examples are only available online)

Example 1. Partch’s 5-limit tonality diamond, after Partch (1974, 110) also given with pitch names and cent deviations from twelve-tone equal temperament

![Diagram of Partch's 5-limit tonality diamond with pitch names and cent deviations](attachment:image.png)
Example 2. Partch’s 11-limit tonality diamond, after Partch (1974, 159)
Example 3. The just-intoned diatonic shown in ratios, cents, and on a Tonnetz

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Cents:</td>
<td>0</td>
<td>204</td>
<td>386</td>
<td>498</td>
<td>702</td>
<td>884</td>
<td>1088</td>
</tr>
</tbody>
</table>

Example 4. Johnston’s accidentals, the 5-limit ration they inflect, the target ratio they bring about, their ratio and cent value. As an example of how this table works, take row 11. If we multiply 4:3 by 33:32 it sums to 11:8. This is equivalent to raising a perfect fourth by Johnston’s 11 chroma.

<table>
<thead>
<tr>
<th>Prime</th>
<th>Starting Ratio Drawn from the 5-limit Lattice</th>
<th>Target Ratio</th>
<th>Accidental Ratio &amp; Overtonal Direction of Inflection</th>
<th>Accidental Approx. cents</th>
<th>Notation Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>40:27</td>
<td>3:2</td>
<td>81:80</td>
<td>up</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>6:5</td>
<td>5:4</td>
<td>25:24</td>
<td>up</td>
<td># b</td>
</tr>
<tr>
<td>7</td>
<td>9:5</td>
<td>7:4</td>
<td>36:35</td>
<td>down</td>
<td>7 7 b 7 b</td>
</tr>
<tr>
<td>11</td>
<td>4:3</td>
<td>11:8</td>
<td>33:32</td>
<td>up</td>
<td>↑ ↓ # b</td>
</tr>
<tr>
<td>13</td>
<td>8:5</td>
<td>13:8</td>
<td>65:64</td>
<td>up</td>
<td>13 13 13 b</td>
</tr>
<tr>
<td>17</td>
<td>25:24</td>
<td>17:16</td>
<td>51:50</td>
<td>up</td>
<td>17 17 17 b</td>
</tr>
<tr>
<td>19</td>
<td>6:5</td>
<td>19:16</td>
<td>96:95</td>
<td>down</td>
<td>19 19 19 19 b</td>
</tr>
<tr>
<td>23</td>
<td>45:32</td>
<td>23:16</td>
<td>46:45</td>
<td>up</td>
<td>23 23 23 b</td>
</tr>
<tr>
<td>29</td>
<td>9:5</td>
<td>29:16</td>
<td>145:144</td>
<td>up</td>
<td>29 29 29 b</td>
</tr>
<tr>
<td>31</td>
<td>15:8</td>
<td>31:16</td>
<td>31:30</td>
<td>up</td>
<td>31 31 31 b</td>
</tr>
</tbody>
</table>
Example 5. The overtone and undertone series of C notated using Johnston’s method

Example 6. A Tonnetz with the syntonic diatonic highlighted in grey. The solid lines connect the two 5-limit pitches that may be inflected to produce a tonal or tonal seventh against the C. This makes clear why the tonal seventh of C is notated as lowered by a syntonic comma in addition to an inverse 7 sign. It is because the pitch is tuned relative to the D- (10:9), which is a syntonic comma lower than the D that appears in the diatonic gamut.
Example 7. Johnston, String Quartet No. 9/I, m. 109. A comma pump progression shown with Roman Numerals and a Tonnetz. The lines connecting circled pitches show the tuning path that leads to C lowered by a syntonic comma.

C+:  I  IV -ii -V (-I)
Example 8. The overall just-intonation form of String Quartet No. 9

Example 9. Johnston, String Quartet No. 9/I, RH A. The overlapping metric scheme of the body of the first movement. Notice that all the pitches of the C diatonic scale are present.
Example 10. Johnston, String Quartet No. 9/I, mm. 11–14. Asserted 3:2 relations are shown on a subset of the 5-limit Tonnetz and on the score with dotted lines.

Example 11. The tonal structure underlying the second movement with rehearsal marks and repeat signs

```
C \| G \rightarrow C \| Am
\[3:2\]
```

```
A \| B \| C
```

```
D \rightarrow E \rightarrow B \rightarrow A \rightarrow C \rightarrow E
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```
F \rightarrow C \rightarrow G \rightarrow D \rightarrow A \rightarrow E
```

```
“Inversion”
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```
A \| G \rightarrow C \| Am \| Dm \rightarrow Am \| 
```

```
\[4:3\]
```

(by assertion)
Example 12. Johnston, Quartet No. 9/II, mm. 1–14

Example 13. The scale seen in Ex. 12, the overtone scale of C (Co17), in ratios and cents with additional marking of the 4:5:6 triads found in the scale
Example 14. Johnston, String Quartet No. 9/II, mm. 15–28. Local tonal centers are indicated beneath the score with ratios in reference to C as 1:1.
Example 15. The modulatory scheme of the B section of the second movement along with this motion on a Tonnetz.
Example 16. Johnston, String Quartet No. 9/II, mm. 29–42
Example 17. The Eu15 scale with the structure of the “dominant” triad, Eu11,13, analyzed
Example 18. Johnston, String Quartet No. 9/II, mm. 43–56 with local tonics indicated beneath the score with their ratios as figured against C as 1:1. These modulations are also shown on a subset of the 5-limit Tonnetz.
Example 20. The triadic motion of the first fourteen measures of the third movement represented on a 5-limit Tonnetz

Example 21. Johnston, String Quartet No. 9/III, mm. 50–51 with the ratio of higher prime passing notes shown beneath the score along with the inverse relationship between the major tonic and minor submediant harmonies
Example 22. Johnston, String Quartet No. 9/III, mm. 57–63 with Roman Numeral analysis integrating Johnston’s syntonic comma notation.
Example 23. The form of String Quartet No. 9/IV with rehearsal marks and tonality in brackets
Example 24. Johnston, String Quartet No. 9/IV, mm. 1–17 with scale resources indicated beneath the score and the modulation to the dominant analyzed using Roman Numerals.
Example 25. Johnston, String Quartet No. 9/IV, mm. 41–52 with scale resources indicated beneath the score and the modulation to the tonic analyzed using Roman Numerals.