Music Theory Online

The Online Journal of the

Society for Music Theory

Volume 9, Number 4, October 2003

Nancy Rogers* and Michael Buchler*

Square Dance Moves and Twelve-Tone Operators: Isomorphisms and New Transformational Models

KEYWORDS: square dance, dance, twelve-tone, transformation, operation, isomorphism, model, Berg, Lulu

ABSTRACT: Both twelve-tone composers and square dance callers use systematic permutations in order to balance variety with familiarity. This paper demonstrates connections between musical and square dance transformations, illustrating some ways in which the two disciplines might inform each other. With nearly seventy moves in the primary or "mainstream" program and a hundred in the more advanced "plus" program, square dance calls could not only augment music theorists' repertoire of transformational devices, but could help expand our fundamental notions of musical transformation. Indeed, non-canonical operations that are considered complex in atonal music theory (such as O'Donnell's split transformations, Mead's O_z, and even Klumpenhouwer's networks) can be modeled by moves that are customary even at the easiest levels of square dance.

Submission received July 2003

[1] If your sole experience with square dancing was in junior high school gym class, then it seems safe to say that you have only seen the tip of the iceberg. About five years ago, unaware that American square dance extended well beyond the "left allemandes," "do-si-dos," and "right-and-left grands" of our childhood, we decided to take lessons. We soon discovered that there are nearly seventy calls in "mainstream" square dancing—the lowest of seven sanctioned skill levels.[1] Square dancers follow calls on a moment-to-moment basis, and keeping track of so many calls can be a substantial mental workout, especially for a beginner.

[2] Square dance instructors typically teach by rote, simply repeating a small number of calls over and over during a class until the corresponding moves become familiar to the new dancers. In the absence of a textbook, we found that the most effective way to memorize a dizzying collection of calls was to diagram new moves immediately after class. Our depictions, we noticed, were remarkably similar to the illustrations one might find in an article on transformational pitch-class set theory. As early as our second lesson, we began focusing particularly on split transformations and cycles. In fact, the music-theoretic connections were so strong that we independently reached the conclusion that men could be seen to represent even-numbered pitch-classes and women could be seen to represent odd-numbered pitch-classes (as will be discussed later).

[3] Disappointingly few square dancers are aware of—let alone interested in—square dancing mechanics, but we found that callers were happy to discuss such matters. Callers were very conscious of the underlying patterns we were observing, although obviously they didn't view these patterns through the lens of twelve-tone theory. We soon learned that callers have their own way of classifying calls according to practical concerns like how long it takes to complete the move and where dancers are positioned at its end. For instance, one caller used the phrase "sixteen-beat zero move" to describe a maneuver in which the dancers end up exactly where they started after sixteen beats.

[4] Callers are clearly analogous to composers, whereas dancers are performers. An experienced square dancer often has a sense of which move is likely to be called next and could bring dancers back to their home positions from a near-by position as effortlessly as most musicians could resolve a dominant seventh chord to the tonic. When the original square has been completely scrambled, however, this process is more like modulating back from the key of the Neapolitan: it takes some skill and training.[2] A good caller can smoothly direct the dancers into unfamiliar positions, completely mixing up the square, and then bring the dancers back home, preferably with a combination of moves so novel that the dancers don't realize they're almost back until the last second. To continue our harmonic analogy, hearing the penultimate structural call (almost always a left allemande with one's corner) is like hearing a cadential $^6_4$ after a remote modulation: no matter how disoriented the dancers may have been, suddenly it is clear that the next move (almost always a right-and-left grand) will be the dominant that takes them home.[3]

We would like to thank our square dance teachers, Naaman Moorehouse and Chuck Lovig, and also express our appreciation to the countless square dancers and callers in eastern Iowa who indirectly supported this project through their dancing and their unfailing encouragement.

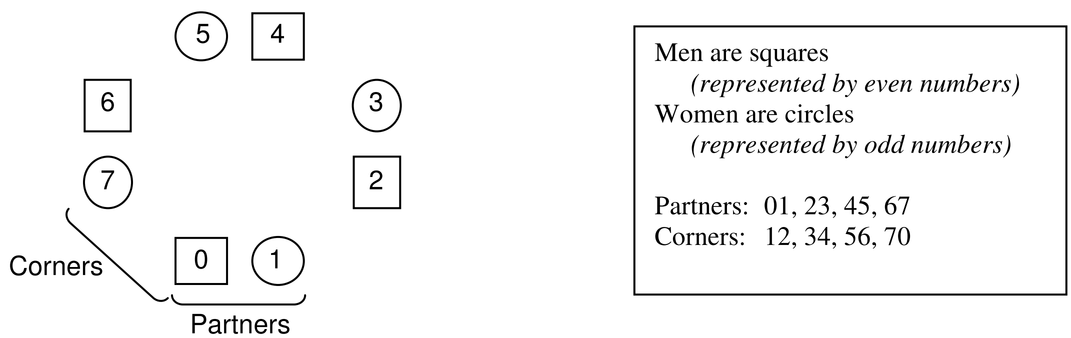
[1] Callerlab is the official sanctioning body of American Square Dance.

[2] In fact, one very experienced caller actually bragged to us that he could return *any* square to home position in only four calls.

[3] Whoops of delight at this point in the dance are common.

[5] More specific comparisons between square dance calling and composition are also quite appropriate. Figure 1 shows the basic starting position for a square dance.[4] Any woman and the man to her left (or any man and the woman to his right) are *partners*; any woman and the man to her right (or any man and the woman to his left) are *corners*. Partners {01} and {45} are designated *head couples* and the others are called *side couples*.[5] The distinction between head and side couples is important solely for practical reasons (e.g., certain moves involve only a pair of couples, so the caller must be able to specify which pair). "Head" should not be construed as connoting any kind of hierarchical superiority; all square dancers are of equal status.

Figure 1. The standard square dance formation and our mod 8 dancer numbers.



[6] From this formation, we can draw some simple parallels to musical transformation. Circling to the right or to the left, for instance, is just like pitch-class transposition. Of course, there are only eight elements in our universe, but the principle is exactly the same. The common square dance call "circle to the right, go half-way 'round" would move every dancer four positions about the square (or T_4), so that everyone would stop exactly half way around the square—a location that we will consider a tritone away. This simple move is animated in Figure 2.[6] Obviously, we could return the dancers to their original positions by asking them to circle four positions either counterclockwise or clockwise (that is, transposition either up or down), or we could use one of several other common calls with the same effect. For instance, successively asking the head couples and side couples to do a "right-and-left through" (depicted in Figure 3) will return all dancers to their original positions.

[7] Square dance calls that are isomorphic with T_n cycles are not remarkable either to dancers or to music theorists, nor are they used as frequently as calls that are isomorphic with multiplication by 5 and its inversion, multiplication by 7, known more simply as M and MI. These are the operations that (respectively) map the chromatic scale onto the circle of fourths or fifths. Of

[4] We have chosen to number dancers according to their order positions because clearly square dance transformations should be understood as applying to the order number domain. No dancer becomes another dancer, obviously; rather, dancers may occupy one another's positions.

[5] Head couples are normally defined as those who face either the caller's end of the room or the opposite end of the room. The two remaining couples, who face to the caller's left and right, are defined as side couples.

[6] Most of our animated figures draw heavily from original animations by square dancer Noriko Takahashi, who graciously allowed us to use them in this article. We are grateful for her assistance, and we encourage readers who are interested in viewing more square dance animations to visit her excellent website (http://members.tripod.com/~noriks/ENGLISH/English-Index2.html).

course, working in our eight-element universe, we must define M as multiplication by 3 (the lowest integer that is relatively prime with 8). As shown in Figure 4, M_3 and M_5 mod 8 are equivalent to M_5 and M_7 mod 12. Notice that we could also view these operations as dual transpositions or dual inversions: M_3 is equivalent to performing T_0I on the even dancers and T_4I on the odd ones, while M_5 is equivalent to performing T_0 on the even dancers and T_4 on the odd ones. "Ladies chain half-way 'round" (more often expressed as "ladies chain straight across")—one of the most common calls in contemporary square dancing—maps dancers by M_5. This move is illustrated in Figure 5.

Figure 4. M and MI ("circle of fifths transform") in both mod 12 and mod 8 universes.

mod 12

M (multiplication by 5)

0	1	2	**3**	4	5	**6**	7	8	**9**	10	11
0	5	10	**3**	8	1	**6**	11	4	**9**	2	7

evens at T_0I; odds at T_6I

MI (multiplication by 7)

0	1	**2**	3	**4**	5	**6**	7	**8**	9	**10**	11
0	7	**2**	9	**4**	11	**6**	1	**8**	3	**10**	5

evens at T_0; odds at T_6

mod 8

M (multiplication by 3)

0	1	2	3	**4**	5	6	7
0	3	6	1	**4**	7	2	5

evens at T_0I; odds at T_4I

MI (multiplication by 5)

0	1	**2**	3	**4**	5	**6**	7
0	5	**2**	7	**4**	1	**6**	3

evens at T_0; odds at T_4

[8] We believe that M and MI, especially when combined with transposition, are preferred because their systematic rotation of the dancers ensures that all men dance with all women within the square. Furthermore, M and MI operations in square dance balance variety with familiarity, just as they do in musical composition. As Andrew Mead observed, "[M and MI] maintain aggregates within the partitions while changing the underlying row's interval pattern in predictable ways."[7] In square dance, of course, it is essential to maintain the aggregate (square dancers do not suddenly wander off!), and if we simply replace the phrase "interval pattern" with "pattern of dancers," then Mead could be describing one of the most common schemas of square dance choreography: while maintaining the same partitions (that is, dance formations), callers change the pattern of dancers in predictable ways. This is especially true of a "singing call," in which the caller quickly inserts calls while crooning a popular tune (frequently retaining a substantial portion of the original lyrics). Given the complexity of singing while calling and the impossibility of rescuing a confused dancer who ends up in the wrong location, singing calls tend to utilize very clear repeated cycles, so that by the end the caller is merely reminding people what to do next.

[9] Like their mod 12 counterparts, the mod 8 cycles of M and MI are each either one or two elements long, meaning that M and MI are their own inverses. However, when one transposes M or MI, cycles of varying lengths are produced. These transpositions and their cycles are shown in Figure 6. For instance, transposing M_3 by 5 (that is, T_5M_3) produces two four-dancer cycles. Callers routinely divide the initial eight-person square into two parallel four-person units, and

[7] Andrew Mead, *An Introduction to the Music of Milton Babbitt*, Princeton, NJ: Princeton University Press (1994), 36.

from these formations all odd-numbered transpositions of M_3 have corresponding common square dance calls in the "circulate" family.[8] Calls to circulate are contextually defined according to the dancers' starting formation (and are therefore difficult to summarize in any detail), but all involve two distinct four-position cycles that maintain the same dance formation. Figure 7 illustrates how calling "circulate" from parallel ocean waves (a configuration discussed in more detail later) produces cycles corresponding to T_5M_3.

Figure 6. The cycles of T_nM and T_nMI mod 8.

Cycles of T_0M: (0), (1,3), (2, 6), (4), (5, 7)

	0	1	2	3	4	5	6	7
maps to:	0	3	6	1	4	7	2	5

Cycles of T_0MI: (0), (1, 5), (2), (3, 7), (4), (6)

	0	1	2	3	4	5	6	7
maps to:	0	5	2	7	4	1	6	3

Cycles of T_1M: (0, 1, 4, 5), (2, 7, 6, 3)

	0	1	2	3	4	5	6	7
maps to:	1	4	7	2	5	0	3	6

Cycles of T_1MI: (0, 1, 6, 7, 4, 5, 2, 3)

	0	1	2	3	4	5	6	7
maps to:	1	6	3	0	5	2	7	4

Cycles of T_2M: (0, 2), (1,5), (3), (4, 6), (7)

	0	1	2	3	4	5	6	7
maps to:	2	5	0	3	6	1	4	7

Cycles of T_2MI: (0, 2, 4, 6) (1, 7, 5, 3)

	0	1	2	3	4	5	6	7
maps to:	2	7	4	1	6	3	0	5

Cycles of T_3M: (0, 3, 4, 7), (1, 6, 5, 2)

	0	1	2	3	4	5	6	7
maps to:	3	6	1	4	7	2	5	0

Cycles of T_3MI: (0, 3, 2, 5, 4, 7, 6, 1)

	0	1	2	3	4	5	6	7
maps to:	3	0	5	2	7	4	1	6

Cycles of T_4M: (0, 4), (1,7), (2), (3, 5), (6)

	0	1	2	3	4	5	6	7
maps to:	4	7	2	5	0	3	6	1

Cycles of T_4MI: (0, 4), (1), (2, 6) (3), (5), (7)

	0	1	2	3	4	5	6	7
maps to:	4	1	6	3	0	5	2	7

Cycles of T_5M: (0, 5, 4, 1), (2, 3, 6, 7)

	0	1	2	3	4	5	6	7
maps to:	5	0	3	6	1	4	7	2

Cycles of T_5MI: (0, 5, 6, 3, 4, 1, 2, 7)

	0	1	2	3	4	5	6	7
maps to:	5	2	7	4	1	6	3	0

Cycles of T_6M: (0, 6), (1), (2, 4), (3, 7), (5)

	0	1	2	3	4	5	6	7
maps to:	6	1	4	7	2	5	0	3

Cycles of T_6MI: (0, 6, 4, 2), (1, 3, 5, 7)

	0	1	2	3	4	5	6	7
maps to:	6	3	0	5	2	7	4	1

Cycles of T_7M: (0, 7, 4, 3), (1, 2, 5, 6)

	0	1	2	3	4	5	6	7
maps to:	7	2	5	0	3	6	1	4

Cycles of T_7MI (0, 7, 2, 1, 4, 3, 6, 5)

	0	1	2	3	4	5	6	7
maps to:	7	4	1	6	3	0	5	2

[10] Similarly, the even transpositions of M_5 have corresponding legitimate square dance calls. T_4M_5 is essentially a tritone transposition of M_5, producing cycles where the odds remain in place while the evens exchange with the opposite dancer. This, of course, is exactly the reverse of T_0M_5 (refer back to Figure 5), and could therefore be expressed as "four men chain half-way 'round."

[8] Such calls include "couples circulate," "box circulate," "diamond circulate," and "ping pong circulate," among others. However, "circulate" may be called generically, and it may be modified to affect only a specified portion of the dancers (for instance, "ladies circulate" or "ends circulate").

T_2M_5 produces one of the most ubiquitous moves in square dancing: the "right-and-left grand".[9] In this move, even and odd dancers rotate separately and in opposite directions, as shown in Figure 8. Under T_6M_5, women move clockwise around the square while men move counterclockwise, producing either the relatively rare "wrong-way grand"—the inverse of the move just mentioned—or the common "eight-chain through." As depicted in Figure 9, however, the latter move begins in the column formation (rather than the square formation appropriate to the "wrong-way grand"). The call "grand weave," while recognized by Callerlab, is not a core component of any sanctioned square dance program and therefore should not used unless the caller trains it first.[10] However, it remains interesting for our purposes because the men's parts feature a complete eight-element T_5M_5 cycle. "Grand weave" is illustrated in Figure 10.

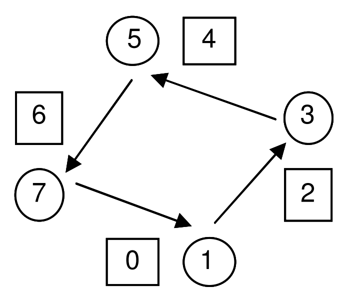
[11] Some common calls produce split transformations that cannot be compared to M and MI or their transpositions. It is very typical for women to cycle counterclockwise around the four sides of the square while men remain in their home positions. In fact, singing calls are almost invariably constructed as an elaboration of this normative formula: a lengthy series of moves returns the men to their home positions while moving the women by T_2; after four iterations, the women have returned to their home positions and the dance concludes. No canonical twelve-tone operator keeps the even elements invariant while mapping the odd elements at T_2, but this instantiates Andrew Mead's O_z function, introduced in his 1988 article on pitch class/order number isomorphisms.[11] Mead's function, which is diagrammed in Figure 11, holds the even pitch classes invariant while cycling the odd ones along their whole-tone scale. We suggest a friendly amendment to the O_z label: Mead defines z as the transposition level applied to even pitch classes, but for our purposes it is more useful to regard O as a function that shifts the odds by T_2 (reflecting motion past a man's position to the next woman's position) and holds the evens invariant. In our O^z, z represents the number of times that O is executed. Our O^1 is the same as Mead's O_2; our O^2 is the same as Mead's O_4, and so forth.

Figure 11. Mead's O^z and its cycles.

O^z holds the even elements invariant while cycling the odd elements about their "whole tone scale" by z places. (As Mead originally defined O_z, odd elements are transposed by z semitones.)

Cycles of O: (0), (2), (4), (6), (1, 3, 5, 7)

Under O^1,	0	1	2	3	4	5	6	7
maps to:	0	3	2	5	4	7	6	1
Under O^2,	0	1	2	3	4	5	6	7
maps to:	0	5	2	7	4	1	6	3
Under O^3,	0	1	2	3	4	5	6	7
maps to:	0	7	2	1	4	3	6	5



[9] It also models the equivalent call "weave the ring," in which dancers follow precisely the same pattern but never touch hands.
[10] One possible reason for the relative obscurity of this move is its extraordinary length. Requiring 64 beats for its execution, "grand weave" is double the length of mainstream square dancing's longest canonical call.
[11] Andrew Mead, "Some Implications of the Pitch Class/Order Number Isomorphism Inherent in the Twelve-Tone System, part I," *Perspectives of New Music* 26:2 (Summer, 1988), 96–163.

[12] As mentioned, frequently an extended series of moves with the net result of O is repeated four times in order to complete the cycle and bring the women back to their home positions. However, the "teacup chain" move from square dancing's "plus" level demonstrates the entire O cycle within a single call.[12] This relatively complicated move is diagrammed in Figure 12a and animated (with step-by-step descriptions) in Figure 12b. To get a good sense of the cyclical motion, we suggest choosing one woman and following her progress around the square while watching the animation; then choose another woman and notice how she follows the same path from a different starting point. The name "teacup chain" is clearly intended to suggest the active involvement of the ladies and the relative exclusion of the gentlemen. A corresponding obscure move known as a "beer mug" reverses the gender roles as well as the rotational direction, but there are also common calls that rotate the men while leaving the women stationary. We therefore suggest a parallel function E^z that holds the odd-numbered dancers stationary while cycling the even-numbered dancers. An illustration of E^{-1} (or E^3), corresponding to the straightforward mainstream call "ladies center while the men sashay," is provided in Figure 13. (We prefer the E^{-1} label because it better represents the motion of the dancers.)

[13] The eight-person square is the most basic square dance formation, but dancers are only in this position for a minority of the time. It serves as a point of departure and arrival, but the square tends to be a comparatively static formation. When it is maintained, the dancers are not moving as much or as creatively as when they are placed into a variety of other alignments. Far more common is the grouping of the dancers into two four-person lines (which have already appeared in Figures 7 and 9).[13] The direction that each dancer faces depends on how the caller led into this formation, and there are many possible combinations since dancers do not necessarily face the same way.

[14] An especially common line-of-four configuration is known as the "ocean wave." In this position, adjacent dancers face opposite directions while touching hands (with women typically–but by no means necessarily—occupying the two interior positions).[14] Although the four-person wave is normative, an "ocean wave" could conceivably contain any number of dancers.[15] The most common call from a wave formation is "swing through," which is animated twice in Figure 14. Swing through is a simple but intriguing move; it produces a single cycle no matter how many dancers are involved. Given any n-dancer ocean wave, n iterations of swing through will form an identity operation, while $n/2$ iterations will form a retrograde operation.[16] (Notice that after two iterations of swing through in Figure 14, the four dancers in the illustration have been retrograded.)

[15] We know of no musical operation—certainly nothing in the canon of twelve-tone operators—that is comparable to swing through.[17] This is not to say, however, that swing

[12] Plus-level square dancing is the skill level immediately above mainstream, involving an additional 32 calls.

[13] These lines may be close together (in which case they are described as "columns") or separated (in which case they are described as "lines").

[14] The designation "ocean wave" refers to the wavy line of the dancers' arms as they touch hands.

[15] Callers will occasionally maneuver all eight dancers into a single wave configuration known as a "tidal wave."

[16] Although square dancers would be quite taken aback by odd-cardinality ocean waves, the formula still applies. Performing half of a swing through is not conceptually difficult, since the move contains two distinct components (i.e., "half by the right" and "half by the left").

[17] We do, however, know of parallels to swing through in the highly permutational world of change ringing. A "Plain Hunt" peal for four bells is equivalent to swing through on a four-person ocean wave. More complex peals that use the type of alternate trading found in swing through include "Plain Bob Minimus" and "Plain Course of Grandshire

through is without musical analog. Consider the three distinct row classes that Berg used to derive the pitch material of his final opera, *Lulu*.[18] When they are transposed to begin on pitch-class 0 (as shown in Figure 15), Dr. Schön's and Alwa's rows have four invariant pitch-classes (in order positions 0, 3, 6, and 9). Dr. Schön's row has been arranged around the perimeter of a four-by-four square in Figure 16 with the invariant pitch-classes in the corners. We'll think of these pitch classes as non-dancing pitch classes, while the remaining eight that form the standard starting square dance configuration are dancing pitch classes. As depicted in Figure 17, the pairs on the North and West sides of the square join to form an ocean wave. The other two couples, following typical square dance practice, likewise join to form an ocean wave. In these parallel waves, the dancers swing through, then step back out into the square formation, transforming Dr. Schön's row into Alwa's.[19]

Figure 15. Three row classes from Alban Berg's *Lulu*.

Main row:	<	0	4	5	2	7	9	6	8	b	a	3	1	>
Dr. Schön's row:	<	0	5	9	a	4	7	6	b	1	2	8	3	>
Alwa's row:	<	0	8	5	a	7	1	6	4	b	2	3	9	>

Figure 16. Dr. Schön's row arranged about a square.

2	1	b	6
8			7
3			4
0	5	9	a

[16] Using a similar (although somewhat more complex) procedure, the main row in *Lulu* can be transformed into Dr. Schön's row. Figure 18 demonstrates how pairs of couples (North and East, South and West) form parallel ocean waves, swing through, and step back into the square. This time, the pitch classes at the corners are not permitted to be wallflowers but instead move at T_0I.[20]

Doubles." All three of these peals are diagrammed in the *Grove Dictionary Online* article on change ringing, and the "Plain Course of Grandshire Doubles" plays an interesting role in Dorothy L. Sayers' 1934 mystery novel *The Nine Tailors*, in which Lord Peter Wimsey illustrates how the peal's abstract structure could be used to decode hidden messages. We thank Lewis Rowell and Mary Wennerstrom for directing us to this most remarkable novel.

[18] These three series are shown in Perle (1985), 94–95 and Headlam (1996), 306 and 397–99.

[19] Dave Headlam also relates the order positions of Dr. Schön's and Alwa's row classes. While he draws many more musical distinctions (especially involving rhythm and set class) than do we, his description of their order-position relationship is less formally defined than our swing-through based transformation: "In Alwa's row <07294B6183A5> [these are order positions relative to the main row], adjacent order-positions differ by 7, and in pairs by 14 = 2 (mod 12); in the Schön row <02591468B37A>, successive order positions differ either by 2 or by combinations of 3s and 4s in 7s. As a result, identical gradations are no more than two notes away in the two rows, and two of the three pitch-classes in each trichord of the rows are identical." (Headlam 1996, 308)

[20] Obviously no existing square dance move can accommodate the motion of ten dancers, as this transformation requires, but there is precedent for large-scale circulating motion from an outside position. Calls such as "load the boat" (from the plus program) rotate the corner dancers 180 degrees in a large semi-circular motion around the inner dancers, who simultaneously perform relatively complicated maneuvers.

[17] The relationship between the main row and Alwa's row can be understood as an application of O^{Z} to the order-number domain: pitch classes in odd-numbered positions are shifted by six positions.[21] This operation, although easily expressed, falls distinctly outside the traditional canon of twelve-tone transformations. Square dancers, however, could easily recognize it as an extension of "ladies chain half-way 'round," using twelve dancers instead of the usual eight. This simple move was depicted from a standard square formation in Figure 5; Figure 19 portrays how it might be applied to a hexagon, thereby transforming Lulu's row into Alwa's.[22]

[18] Moving among Berg's row forms requires non-canonical operations that are considered complex in atonal music theory. As we have seen, however, dual operations such as Mead's O^{Z} can be modeled by moves that are customary even at the easiest levels of square dance.[23] Indeed, the structure of any square dance could easily inspire a host of new compositional and theoretical constructs. We have only discussed relatively straightforward calls that begin and end in the same basic configuration, omitting the many calls that lead the dancers between different configurations (e.g., converting the basic square to columns).[24] Furthermore, in comparing square dance calls to musical transformation, we have actually simplified square dance considerably by omitting parameters such as the facing direction of each dancer — a critical factor, because some calls are impossible if dancers are facing the wrong direction, while others are interpreted differently depending on the dancers' facing direction. Additional parameters could, we speculate, serve as very interesting compositional source material; the direction each dancer faces, for instance, certainly suggests comparisons with voice leading.

[19] Some of the connections that we have suggested between square dancing and twelve-tone theory are only analogies (e.g., callers are like composers, the right-and-left grand is like a cadential dominant, and the distance across a square is like a tritone). Most of the square dance moves we have presented, however, would be better described as instantiations of permutational theory as applied to an eight-element universe. While describing the dancers' paths in mathematical formulas fails to capture nuances such as facing direction and arm motions, we firmly believe that expressing square dance moves as mappings with cyclical properties reflects a mode of thought

---

[21] Adopting Mead's convention of designating order-number operations in boldface type, one might notate this as $\mathbf{O}^{\mathbf{3}}$. One can also transform the main row to Alwa's row using the more standard M_7 in the order position domain. This is the approach taken by Dave Headlam (1996, 307).

[22] When we initially proposed this model, we were completely unaware that six-couple square dancing is occasionally practiced by experienced dancers. Since square dance moves are defined in terms of four couples, for practical reasons the concept of chaining half-way around may be understood differently from our portrayal in Figure 19. The chair of Callerlab's definitions committee, Clark Baker, explains how standard four-couple calls can be successfully interpreted by six couples in his intriguing 2002 article "Hexagon Squares" (http://www.tiac.net/~mabaker/hexagon.html). We would like to thank Mr. Baker for providing us with a variety of helpful information.

[23] Other dual operations that model square dance moves include O'Donnell's split transformations and Klumpenhouwer's networks. In fact, any split (dual) transformation (where the two parts of the set map differently) can be modeled using isographic Klumpenhouwer networks (K-nets), and the sorts of symmetrical dual operations described in this article would particularly lend themselves to positively isographic K-nets. The abstract and highly general nature of K-nets would, however, blur many of the distinctions among square dance moves that we have tried to draw.

[24] Moves that serve almost as transitions between different kinds of space are quite interesting, but they dramatically complicate the notion of order position. Should dancer 0 serve as a defining reference point? Given that various transitional calls will locate him differently in the new configuration, should one instead arbitrarily choose a new position to designate 0? Perhaps we should view the dancers as eight elements moving over a field of sixteen positions (that is, a four-by-four square). Such questions are well worth exploring, but fall beyond the scope of this paper.

actively used by successful callers. Indeed, some callers discuss their craft with a level of abstraction (as well as a reliance on mathematical concepts) that would surprise many non-dancing music theorists.[25] To dismiss square dance as a social phenomenon with no deeper organizing principles is like dismissing music as a social phenomenon with no deeper organizing principles.

[20] When we started square dancing, we were amused that our knowledge of transformational theory seemed to facilitate our learning and enrich our understanding of square dance. Only later did we realize the degree to which square dance, in turn, can not only enrich our understanding of transformational theory, but can also provide new constructs that seem musically relevant. Our point here is not merely to demonstrate correspondences between theoretical and musical constructs and events that occur during a typical square dance. There are times when a music theorist needs to call upon (or create) some rather complex transformations to explain or model a musical passage. As curious a source as it might seem, square dance provides a substantial and largely uncategorized repository of transformations that can be musically applied.

Return to beginning of article

[25] Consider the quotes below, for instance (all of which are drawn from Clark Baker's "Hexagon Squares"):

> "The claim above (every 4-couple motion has a well-defined 6-couple motion) can be backed up solidly by giving the mapping from one motion to the other. This is done most naturally in polar coordinates. If a dancer in a 4-couple square moves from (a0, r0, d0) to (a1, r1, d1) where a0 and a1 are the polar angles around the center of the square, r0 and r1 are the distances from the center of the square, and d0 and d1 are the dancer's starting and final facing directions, then the equivalent 6-couple dancer's motion would change by the following: ((a1-a0)*2/3, r1-r0, (d1-d0)-(a1-a0)*1/3)."
>
> — Justin Legakis

> "To eliminate the running around, draw the half plane that you are dancing on on a sheet of rubber, and stretch it so that the two sides of the boundary line join each other. Mathematically, if the line is the x-axis, just double the value of theta."
>
> — Andy Latto

> "All this works because no-one ever touches the center of the set, so square dancing is really done on the punctured plane. Dancing with 8, 12, or 4N dancers is lifting the motion of the dancers to a covering space. These are the only covering spaces of the punctured plane, so there's no way to extend this hack further. If only there was a second spot that no dancer ever touched, the fundamental group would be much bigger, so there would be many more covering spaces, and many more strange ways to dance."
>
> — Andy Latto

> "For those interested in such things, I think trying to develop a Hex resolution system is a great exercise. For many of us, sight resolution systems came to us fully developed. They are presented as an algorithm that we have to memorize and follow and later we will be able to expand on the system. In Hex you have to start from scratch."
>
> — Clark Baker

As the homepage for the Tech Squares (MIT's square and round dance club) asks, "When was the last time group theory got your blood moving?"

Works Cited

American Square Dance Society. 1983. *The Plus Movements of Square Dancing*. Handbook series, reprinted from *Square Dancing*: *the Official Publication of the Sets in Order*.

———. 1987. *The Illustrated Basic and Mainstream Movements of Square Dancing*. Handbook series, reprinted from *Square Dancing: the Official Publication of the Sets in Order.*

Baker, Clark M. 2002. "Hexagon Squares." http://www.tiac.net/~mabaker/hexagon.html.

Headlam, Dave. 1996. *The Music of Alban Berg*. New Haven: Yale University Press.

Mead, Andrew W. 1988. "Some Implications of the Pitch-Class/Order-Number Isomorphism Inherent in the Twelve-Tone System: Part One." *Perspectives of New Music* 26/2: 96-163.

———. 1994. *An Introduction to the Music of Milton Babbitt.* Princeton, NJ: Princeton University Press.

Morris, Robert D. 1987. *Composition with Pitch Classes.* New Haven: Yale University Press.

Perle, George. 1985. *The Operas of Alban Berg: Volume II—Lulu*. Berkeley: University of California Press.

Sayers, Dorothy L. 1934. *The Nine Tailors.* New York: Harcourt, Brace.

Wilson, Wilfrid G. and Steve Coleman. 2003. "Change ringing," *The New Grove Dictionary of Music and Musicians Online*, ed. L. Macy (Accessed 30 June 2003), http://www.grovemusic.com.

Valuable square dance web sites:

http://www.dosado.com/default.htm
http://www.callerlab.org

For additional animations of square dance moves, see:

http://members.tripod.com/~noriks/ENGLISH/English-Index2.html
http://www.squaredancecd.com

Return to beginning of article

Nancy M. Rogers
Florida State University
School of Music
Tallahassee, FL 32306-1180
Nancy.Rogers@music.fsu.edu

Michael H. Buchler
Florida State University
School of Music
Tallahassee, FL 32306-1180
Michael.Buchler@music.fsu.edu

Return to beginning of article

Return to beginning of article

prepared by
Brent Yorgason, Managing Editor
Updated 22 October 2003