



MTO 22.4 Examples: Gotham and Gunn, Pitch Properties of the Pedal Harp

(Note: audio, video, and other interactive examples are only available online)

<http://www.mtosmt.org/issues/mto.16.22.4/mto.16.22.4.gotham.php>

Table 1.

1a. All pitch-class sets of cardinalities from 4 to 7, along with the prime forms for those that can be reached on the pedal harp, and the number of different pedal settings that generate them.

1b. The pitch-class sets of Table 1a, listed by order of the number of different pedal settings that generate them (from most to least).

Prime form	Forte name	Interval Vector	Number	Prime form	Forte name	Interval Vector	Number
	4-1	<321000>		[0,1,3,5,7,9]	6-34	<142422>	90
	4-2	<221100>		[0,2,3,5,7,9]	6-33	<143241>	80
	4-3	<212100>		[0,1,3,5,8,9]	6-31	<223431>	78
	4-4	<211110>		[0,1,4,6,9]	5-32	<113221>	56
	4-5	<210111>		[0,1,2,4,7,9]	6-Z47	<233241>	50
	4-6	<210021>		[0,2,4,7,9]	5-35	<032140>	45
	4-7	<201210>		[0,1,3,6,8]	5-29	<122131>	44
	4-8	<200121>		[0,1,3,6,9]	5-31	<114112>	44
	4-9	<200022>		[0,1,2,5,6,9]	6-Z44	<313431>	44
	4-10	<122010>		[0,1,3,7,8]	5-20	<211231>	40
	4-11	<121110>		[0,1,3,5,6,8]	6-Z25	<233241>	40
	4-12	<112101>		[0,1,3,4,6,9]	6-27	<225222>	40
	4-13	<112011>		[0,1,4,6,7,9]	6-Z50	<224232>	40
	4-14	<111120>		[0,1,3,6,7,9]	6-30	<224223>	40
	4-Z15	<111111>		[0,1,2,4,7,8,9]	7-20	<433452>	40
	4-Z29	<111111>		[0,1,2,5,7,8]	6-18	<322242>	38
	4-16	<110121>		[0,2,4,5,7,9]	6-32	<143250>	38
	4-17	<102210>		[0,1,3,4,6,8,9]	7-32	<335442>	38
	4-18	<102111>		[0,1,3,5,8]	5-27	<122230>	36
[0,1,4,8]	4-19	<101310>	4	[0,1,3,6,8,9]	6-Z29	<224232>	36
[0,1,5,8]	4-20	<101220>	8	[0,1,2,4,6,8,9]	7-30	<343542>	36
	4-21	<030201>		[0,1,2,5,7,9]	6-Z48	<232341>	35
	4-22	<021120>		[0,1,4,6,8]	5-30	<121321>	34
	4-23	<021030>		[0,1,3,5,6,7,9]	7-28	<344433>	34
[0,2,4,8]	4-24	<020301>	1	[0,1,2,4,6,7,9]	7-29	<344352>	34
	4-25	<020202>		[0,2,4,6,9]	5-34	<032221>	33
[0,3,5,8]	4-26	<012120>	10	[0,2,3,6,8]	5-28	<122212>	32
[0,2,5,8]	4-27	<012111>	16	[0,1,2,4,6,9]	6-Z46	<233331>	32
[0,3,6,9]	4-28	<004002>	3	[0,1,3,4,6,7,9]	7-31	<336333>	32
	5-1	<432100>		[0,1,2,5,6,8]	6-Z43	<322332>	30
	5-2	<332110>		[0,1,3,4,7,9]	6-Z49	<224322>	30
	5-3	<322210>		[0,1,2,4,5,8,9]	7-21	<424641>	30
	5-4	<322111>		[0,1,3,5,6,9]	6-Z28	<224322>	29
	5-5	<321121>		[0,2,3,5,8]	5-25	<123121>	28
	5-6	<311221>		[0,1,3,4,7,8]	6-Z19	<313431>	28
	5-7	<310132>		[0,1,2,4,7,8]	6-Z17	<322332>	26
	5-8	<232201>		[0,1,3,4,6,8]	6-Z24	<233331>	26
	5-9	<231211>		[0,1,3,5,7,8]	6-Z26	<232341>	26
	5-10	<223111>		[0,1,4,5,8]	5-21	<202420>	24
	5-11	<222220>		[0,1,2,4,6,8]	6-22	<241422>	24
	5-Z12	<222121>		[0,2,3,4,6,7,9]	7-25	<345342>	24
	5-Z36	<222121>		[0,1,3,4,5,7,9]	7-26	<344532>	24
[0,1,2,4,8]	5-13	<221311>	2	[0,1,2,4,5,7,9]	7-27	<344451>	24
	5-14	<221131>		[0,2,4,5,8]	5-26	<122311>	22
	5-15	<220222>		[0,1,2,3,6,8]	6-Z41	<332232>	22
	5-16	<213211>		[0,1,2,3,5,8,9]	7-Z18	<434442>	22
[0,1,3,4,8]	5-Z17	<212320>	4	[0,1,2,3,6,7,9]	7-19	<434343>	22
[0,3,4,5,8]	5-Z37	<212320>	7	[0,1,2,5,6,8,9]	7-22	<424542>	21
	5-Z18	<212221>		[0,1,2,3,6,7,8]	7-7	<532353>	20
[0,1,2,5,8]	5-Z38	<212221>	14	[0,1,3,5,6,8,10]	7-35	<254361>	20
	5-19	<212122>		[0,1,4,5,6,8]	6-16	<322431>	18
[0,1,3,7,8]	5-20	<211231>	40	[0,1,3,4,6,8,10]	7-34	<254442>	17
[0,1,4,5,8]	5-21	<202420>	24	[0,2,5,8]	4-27	<012111>	16
[0,1,4,7,8]	5-22	<202321>	16	[0,1,4,7,8]	5-22	<202321>	16
	5-23	<132130>		[0,1,2,4,5,8]	6-15	<323421>	16
	5-24	<131221>		[0,1,2,4,5,7,8]	7-Z38	<434442>	16
[0,2,3,5,8]	5-25	<123121>	28	[0,2,3,4,5,7,9]	7-23	<354351>	16
[0,2,4,5,8]	5-26	<122311>	22	[0,1,2,3,5,7,9]	7-24	<353442>	16
[0,1,3,5,8]	5-27	<122230>	36	[0,2,3,4,6,9]	6-Z45	<232422>	15
[0,2,3,6,8]	5-28	<122212>	32	[0,1,2,4,6,8,10]	7-33	<262623>	15
[0,1,3,6,8]	5-29	<122131>	44	[0,1,2,5,8]	5-Z38	<212221>	14
[0,1,4,6,8]	5-30	<121321>	34	[0,2,4,6,8]	5-33	<040402>	14
[0,1,3,6,9]	5-31	<114112>	44	[0,1,2,3,5,8]	6-Z40	<333231>	14
[0,1,4,6,9]	5-32	<113221>	56	[0,1,3,4,5,8]	6-14	<323430>	14
[0,2,4,6,8]	5-33	<040402>	14	[0,1,2,3,5,7,8]	7-14	<443352>	14
[0,2,4,6,9]	5-34	<032221>	33	[0,1,2,3,7,8]	6-Z38	<421242>	12
[0,2,4,7,9]	5-35	<032140>	45	[0,1,4,5,8,9]	6-20	<303630>	12
	6-1	<543210>		[0,2,3,4,6,8]	6-21	<242412>	12
	6-2	<443211>		[0,3,5,8]	4-26	<012120>	10
	6-Z3	<433221>		[0,2,3,5,6,8]	6-Z23	<234222>	10
	6-Z36	<433221>		[0,1,2,4,5,6,8]	7-13	<443532>	10
	6-Z4	<432321>		[0,1,2,4,6,7,8]	7-15	<442443>	10
[0,1,2,3,4,8]	6-Z37	<432321>	1	[0,2,4,6,8,10]	6-35	<060603>	9
	6-5	<422232>		[0,1,5,8]	4-20	<101220>	8
[0,1,2,3,7,8]	6-Z38	<421242>	12	[0,2,3,4,5,8]	6-Z39	<333321>	8
	6-7	<420243>		[0,1,2,3,6,9]	6-Z42	<324222>	8
	6-8	<343230>		[0,1,3,4,5,6,8]	7-11	<444441>	8
	6-9	<342231>		[0,1,2,3,5,6,8]	7-Z36	<444342>	8
	6-Z10	<333321>		[0,1,2,3,5,6,9]	7-16	<435432>	8
[0,2,3,4,5,8]	6-Z39	<333321>	8	[0,3,4,5,8]	5-Z37	<212320>	7
	6-Z11	<333231>		[0,1,3,4,5,7,8]	7-Z37	<434541>	7
[0,1,2,3,5,8]	6-Z40	<333231>	14	[0,1,2,4,5,6,9]	7-Z17	<434541>	5
	6-Z12	<332232>		[0,1,4,8]	4-19	<101310>	4
[0,1,2,3,6,8]	6-Z41	<332232>	22	[0,1,3,4,8]	5-Z17	<212320>	4
	6-Z13	<324222>		[0,3,6,9]	4-28	<004002>	3
[0,1,2,3,6,9]	6-Z42	<324222>	8	[0,1,2,4,8]	5-13	<221311>	2
[0,1,3,4,5,8]	6-14	<323430>	14	[0,1,2,3,4,7,8]	7-6	<533442>	2
[0,1,2,4,5,8]	6-15	<323421>	16	[0,1,2,3,4,6,8]	7-9	<453432>	2
[0,1,4,5,6,8]	6-16	<322431>	18	[0,1,2,3,4,6,9]	7-10	<445332>	2
[0,1,2,4,7,8]	6-Z17	<322332>	26	[0,2,4,8]	4-24	<020301>	1
[0,1,2,5,6,8]	6-Z43	<322332>	30	[0,1,2,3,4,8]	6-Z37	<432321>	1
[0,1,2,5,7,8]	6-18	<322242>	38	[0,2,3,4,5,6,8]	7-8	<454422>	1
[0,1,3,4,7,8]	6-Z19	<313431>	28	[0,1,2,3,4,7,9]	7-Z12	<444342>	1
[0,1,2,5,6,9]	6-Z44	<313431>	44				
[0,1,4,5,8,9]	6-20	<303630>	12				
[0,2,3,4,6,8]	6-21	<242412>	12				
[0,1,2,4,6,8]	6-22	<241422>	24				
[0,2,3,5,6,8]	6-Z23	<234222>	10				
[0,2,3,4,6,9]	6-Z45	<234222>	15				
[0,1,3,4,6,8]	6-Z24	<233331>	26				
[0,1,2,4,6,9]	6-Z46	<233331>	32				
[0,1,3,5,6,8]	6-Z25	<233241>	40				
[0,1,2,4,7,9]	6-Z47	<233241>	50				
[0,1,3,5,7,8]	6-Z26	<232341>	26				
[0,1,2,5,7,9]	6-Z48	<232341>	35				
[0,1,3,4,6,9]	6-27	<225222>	40				
[0,1,3,5,6,9]	6-Z28	<224322>	29				
[0,1,3,4,7,9]	6-Z49	<224322>	30				
[0,1,3,6,8,9]	6-Z29	<224232>	36				
[0,1,4,6,7,9]	6-Z50	<224232>	40				
[0,1,3,6,7,9]	6-30	<224223>	40				
[0,1,3,5,8,9]	6-31	<223431>	78				
[0,2,4,5,7,9]	6-32	<143250>	38				
[0,2,3,5,7,9]	6-33	<143241>	80				
[0,1,3,5,7,9]	6-34	<142422>	90				
[0,2,4,6,8,10]	6-35	<060603>	9				
	7-1	<654321>					
	7-2	<554331>					
	7-3	<544431>					
	7-4	<544332>					
	7-5	<543342>					
[0,1,2,3,4,7,8]	7-6	<533442>	2				
[0,1,2,3,6,7,8]	7-7	<532353>	20				
[0,2,3,4,5,6,8]	7-8	<454422>	1				
[0,1,2,3,4,6,8]	7-9	<453432>	2				
[0,1,2,3,4,6,9]	7-10	<445332>	2				
[0,1,3,4,5,6,8]	7-11	<444441>	8				
[0,1,2,3,4,7,9]	7-Z12	<444342>	1				
[0,1,2,3,5,6,8]	7-Z36	<444342>	8				
[0,1,2,4,5,6,8]	7-13	<443532>	10				
[0,1,2,3,5,7,8]	7-14	<443352>	14				
[0,1,2,4,6,7,8]	7-15	<442443>	10				
[0,1,2,3,5,6,9]	7-16	<435432>	8				
[0,1,2,4,5,6,9]	7-Z17	<434541>	5				
[0,1,3,4,5,7,8]	7-Z37	<434541>	7				
[0,1,2,3,5,8,9]	7-Z18	<434442>	22				
[0,1,2,4,5,7,8]	7-Z38	<434442>	16				
[0,1,2,3,6,7,9]	7-19	<434343>	22				
[0,1,2,4,7,8,9]	7-20	<433452>	40				
[0,1,2,4,5,8,9]	7-21	<424641>	30				
[0,1,2,5,6,8,9]	7-22	<424542>	21				
[0,2,3,4,5,7,9]	7-23	<354351>	16				
[0,1,2,3,5,7,9]	7-24	<353442>	16				
[0,2,3,4,6,7,9]	7-25	<345342>	24				
[0,1,3,4,5,7,9]	7-26	<344532>	24				
[0,1,2,4,5,7,9]	7-27	<344451>	24				
[0,1,3,5,6,7,9]	7-28	<344433>	34				
[0,1,2,4,6,7,9]	7-29	<344352>	34				
[0,1,2,4,6,8,9]	7-30	<343542>	36				
[0,1,3,4,6,7,9]	7-31	<336333>	32				
[0,1,3,4,6,8,9]	7-32	<335442>	38				
[0,1,2,4,6,8,10]	7-33	<262623>	15				
[0,1,3,4,6,8,10]	7-34	<254442>	17				
[0,1,3,5,6,8,10]	7-35	<254361>	20				
	Total:		2187				

Table 2.

2a. A list of sets obtainable as subsets of available pedal settings, with a sample set of pitches for each (between D and B strings, including a “-” for any internal strings excluded).

2b. A list of those sets that are not obtainable even as subsets.

Forte name	Prime form	Example	Forte name	Prime form
4-1	[0123]	D#EFGb	4-6	[0127]
4-2	[0124]	D#EFG	4-9	[0167]
4-3	[0134]	D#EF#G		
4-4	[0125]	D#EFG#	5-1	[01234]
4-5	[0126]	DEbFbG#		
4-7	[0145]	DEbF#G	6-1	[012345]
4-8	[0156]	EbFbG#A	6-2	[012346]
4-10	[0235]	DEFG	6-Z3	[012356]
4-11	[0135]	DEbFG	6-Z36	[012347]
4-12	[0236]	DEFG#	6-Z4	[012456]
4-13	[0136]	DEbFG#		
4-14	[0237]	DEbFG#	7-1	[0123456]
4-Z15	[0146]	DEbF#G#	7-2	[0123457]
4-Z29	[0137]	DEbF-A	7-3	[0123458]
4-16	[0157]	DEb-GA	7-4	[0123467]
4-17	[0347]	D-FGbA	7-5	[0123567]
4-18	[0147]	DEbF#-A		
4-21	[0246]	DEF#G#		= 13 total
4-22	[0247]	DEF#-A		
4-23	[0257]	DE-GA		
4-25	[0268]	DE-G#A#		
5-2	[01235]	D#EFGbAb		
5-3	[01245]	D#EFGAb		
5-4	[01236]	D#EFGbA		
5-5	[01237]	D#EFGbA#		
5-6	[01256]	D#EFG#A		
5-7	[01267]	DEbFbG#A		
5-8	[02346]	DEFGbAb		
5-9	[01246]	DEbFbGbAb		
5-10	[01346]	DEbFGbAb		
5-11	[02347]	DEFGbA		
5-Z12	[01356]	DEbFGAb		
5-Z36	[01247]	DEbFbGbA		
5-14	[01257]	DEbFbGA		
5-15	[01268]	DEbFbG#A#		
5-16	[01347]	DEbFGbA		
5-Z18	[01457]	DEbF#GA		
5-19	[01367]	DEbFG#A		
5-23	[02357]	DEFGA		
5-24	[01357]	DEbFGA		
6-5	[012367]	D#EFGbABb		
6-Z6	[012567]	D#EFG#ABb		
6-7	[012678]	DEbFbG#ABb		
6-8	[023457]	DEFGbAbBb		
6-9	[012357]	D#EFGbAbBb		
6-Z10	[013457]	D#EF#GAbBb		
6-Z11	[012457]	D#EFGAbBb		
6-Z12	[012467]	D#EFGABb		
6-Z13	[013467]	D#EF#GABb		

Figure 1. Four plausible scales derived from the pitch-class set 6-34. Modifications are shown by “-” for missing scale degrees, and # or ♭ for chromatic alterations.

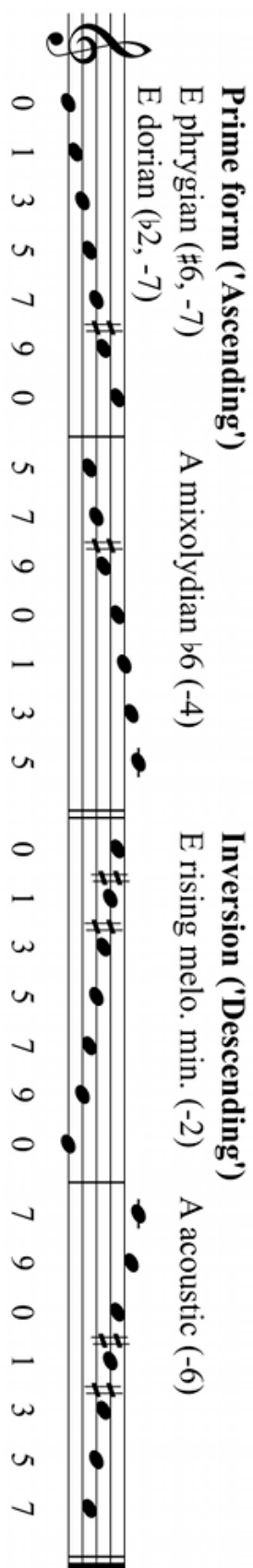


Table 3. Cardinality data: the total number of pitch-class sets of each cardinality; the number that can be realized on the harp, including subsets of pedal configurations; and the number that can be realized by the full pitch collection of a given pedal setting alone.

Cardinality	0	1	2	3	4	5	6	7	8	9	10	11	12
Total Ordinals	1	1	6	12	29	38	50	38	29	12	6	1	1
Harp inc. subsets	1	1	6	12	27	37	45	33	0	0	0	0	0
Harp, pedal only	0	0	0	0	6	18	36	33	0	0	0	0	0

Figure 2. A bar graph corresponding to the data in Table 3

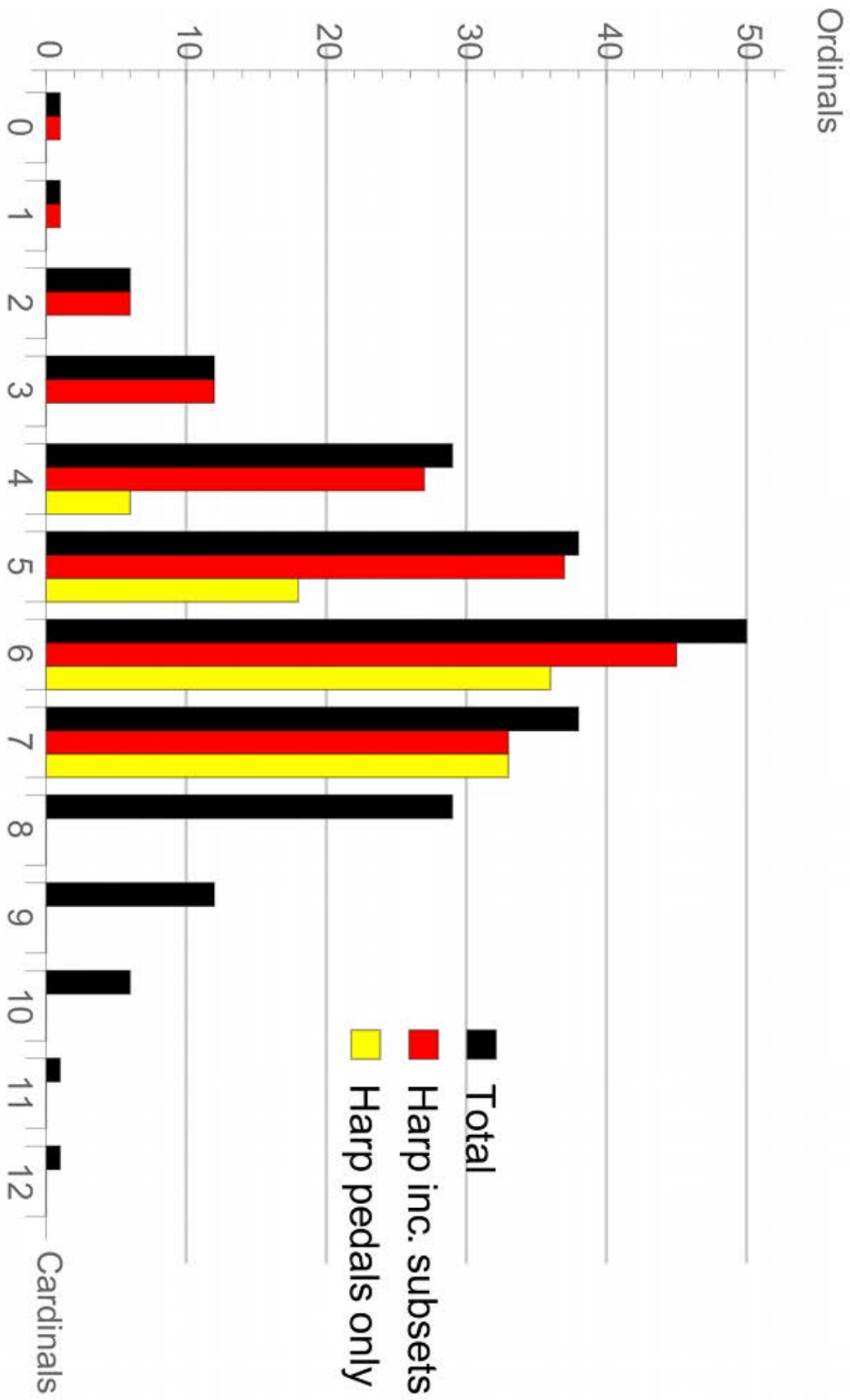


Figure 3a. Pedal changes from $C\flat$ major: each pedal may move up only, leaving 7 possible changes

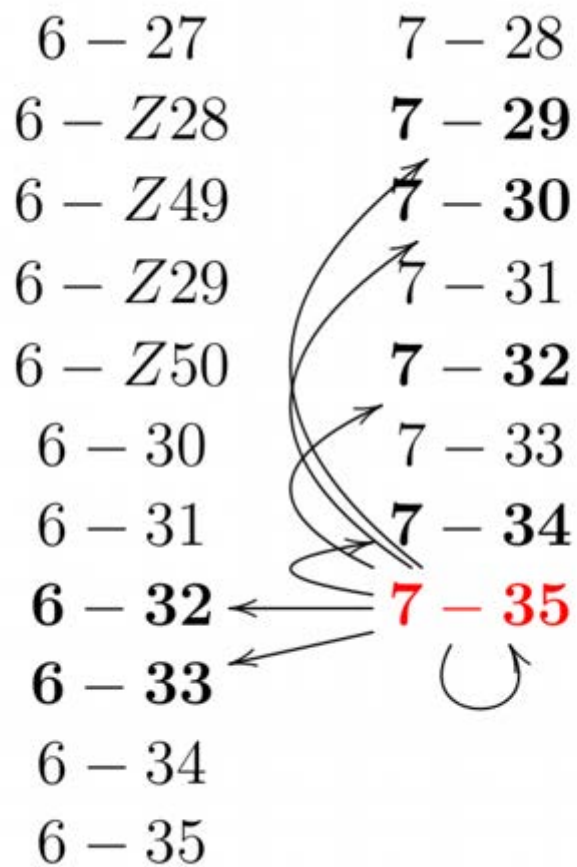


Figure 3b. Pedal changes from C major: each pedal can move up or down, so 14 changes are possible, though they are paired, leading to the same 7 pitch-class sets (twice each).

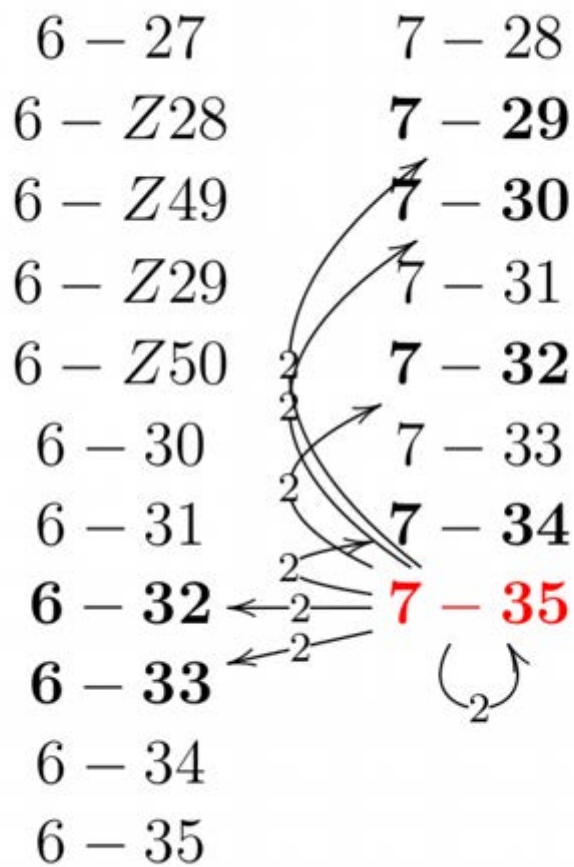


Table 4. A “pedal space” corresponding to harp pedal positions, with numbers of sharps plotted against numbers of flats and the number of pedal settings given for each grid square.

Flats	7	1							
	6	7	7						
	5	21	42	21					
	4	35	105	105	35				
	3	35	140	210	140	35			
	2	21	105	210	210	105	21		
	1	7	42	105	140	105	42	7	
	0	1	7	21	35	35	21	7	1
		0	1	2	3	4	5	6	7

Sharps

Figure 4. Three illustrative excerpts from the first movement of Hindemith's Sonata for Harp (1939)

1 m. 1

2 m. 15

3 [End]

fa # f
si ♯ h
la ♯ a
re ♯ d
do # c

mi ♯ e
fa # f
do ♭ c
la ♯ a
re ♭ d
sol ♭ g

pp

Table 5. An internal sub-grid extending the pedal space. This 2-dimensional grid plots 1 sharp against 1 flat to occupy the relevant grid square { 1,1 } on the harp pedal space of Table 4.

	F#	C#	G#	D#	A#	E#	B#
Bb							/
Eb						/	
Ab					/		
Db				/			
Gb			/				
Cb		/					
Fb	/						

Table 6. The start of a basic Hamiltonian path among all pedal settings.

C	D	E	F	G	A	B
b	b	b	b	b	b	b
b	b	b	b	b	b	nat
b	b	b	b	b	b	#
b	b	b	b	b	nat	#
b	b	b	b	b	nat	nat
b	b	b	b	b	nat	b
b	b	b	b	b	#	b
b	b	b	b	b	#	nat
b	b	b	b	b	#	#
b	b	b	b	nat	#	#
b	b	b	b	nat	#	nat
b	b	b	b	nat	#	b
b	b	b	b	nat	nat	b
b	b	b	b	nat	nat	nat
b	b	b	b	nat	nat	#
b	b	b	b	nat	b	#
b	b	b	b	nat	b	nat
b	b	b	b	nat	b	b
b	b	b	b	#	b	b
b	b	b	b	#	b	nat
b	b	b	b	#	b	#
b	b	b	b	#	nat	#
b	b	b	b	#	nat	nat
b	b	b	b	#	nat	b
b	b	b	b	#	#	b
b	b	b	b	#	#	nat
b	b	b	b	#	#	#

... cont.