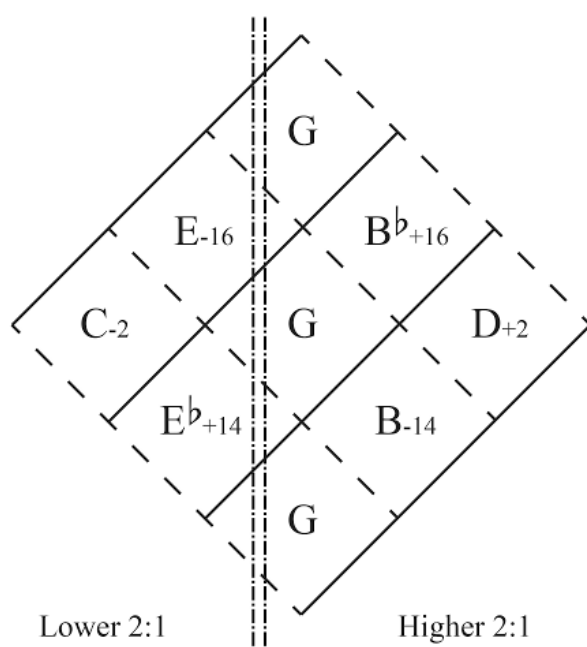
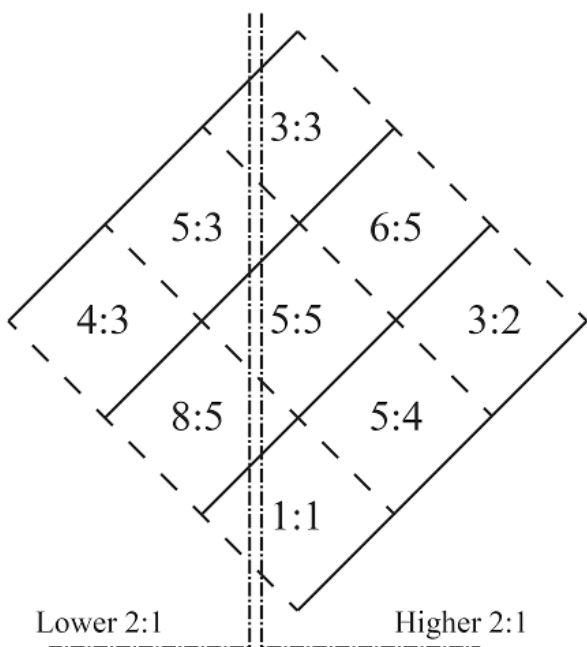


**MTO 25.1 Examples: Willis, Comprehensibility and Ben Johnston's String Quartet No. 9**

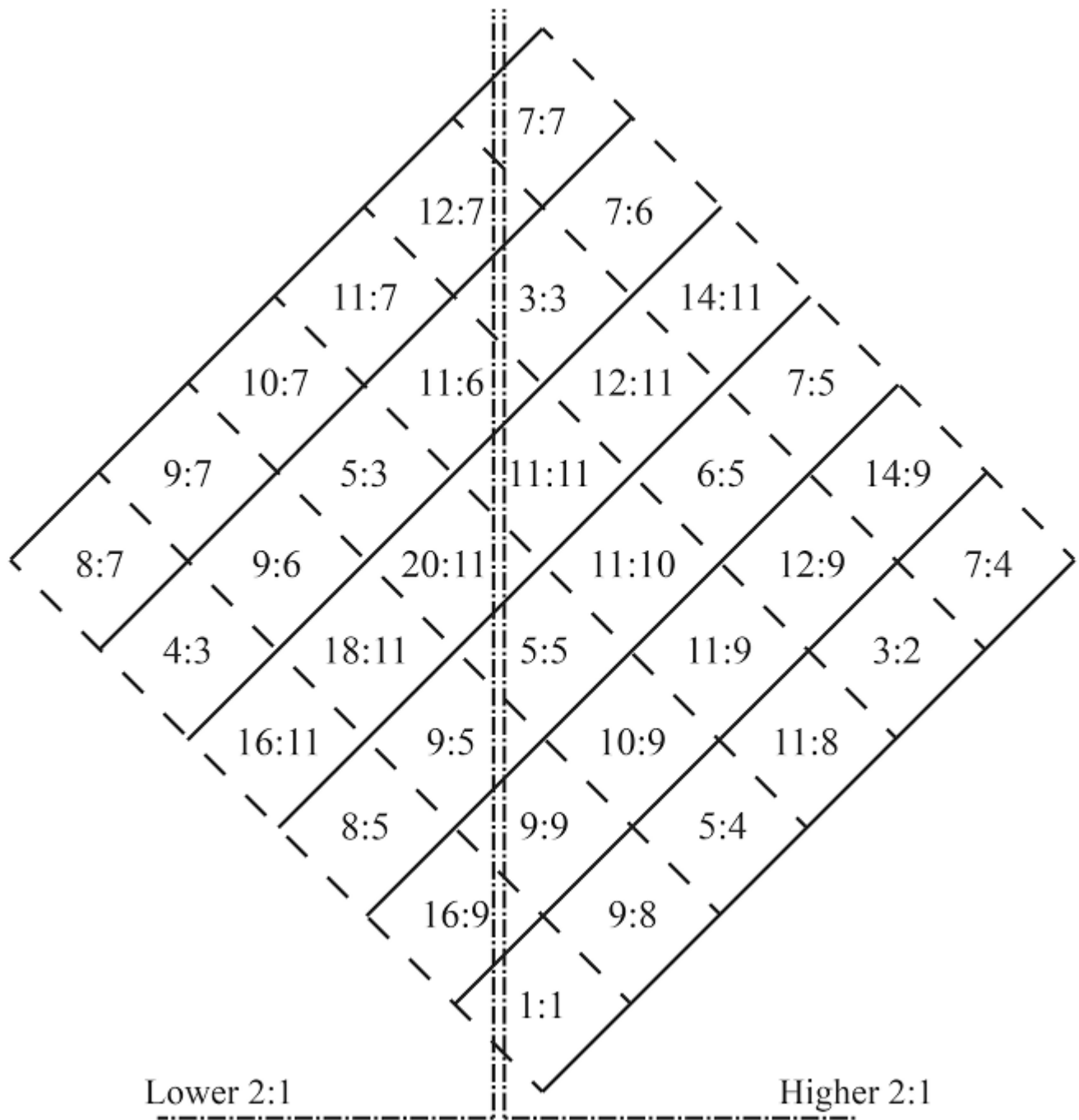
(Note: audio, video, and other interactive examples are only available online)

<http://mtosmt.org/issues/mto.19.25.1/mto.19.25.1.willis.html>

**Example 1.** Partch's 5-limit tonality diamond, after Partch (1974, 110) also given with pitch names and cent deviations from twelve-tone equal temperament

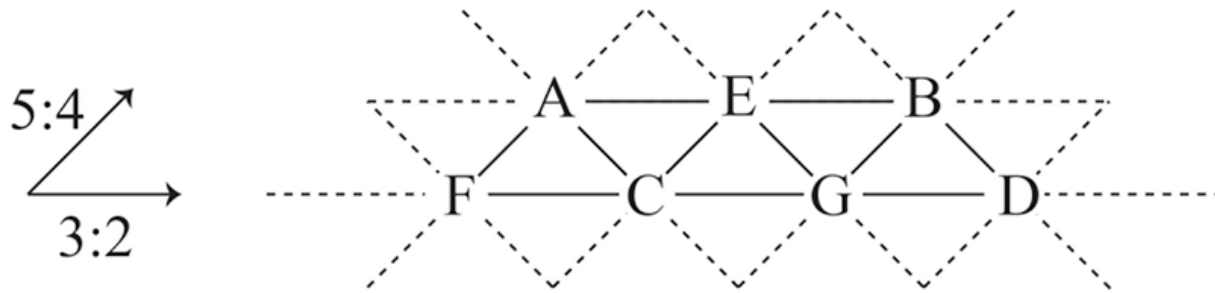


Example 2. Partch's 11-limit tonality diamond, after Partch (1974, 159)



**Example 3.** The just-intoned diatonic shown in ratios, cents, and on a *Tonnetz*

Ratio: C(1:1) D(9:8) E(5:4) F(4:3) G(3:2) A(5:3) B(15:8)  
 Cents: 0 204 386 498 702 884 1088



**Example 4.** Johnston's accidentals, the 5-limit ration they inflect, the target ratio they bring about, their ratio and cent value. As an example of how this table works, take row 11. If we multiply 4:3 by 33:32 it sums to 11:8. This is equivalent to raising a perfect fourth by Johnston's 11 chroma.

Prime	Starting Ratio Drawn from the 5-limit Lattice	Target Ratio	Accidental Ratio & Overtone Direction of Inflection		Accidental Approx. cents	Notation Examples
3	40:27	3:2	81:80	up	21.5	+ -
5	6:5	5:4	25:24	up	70.7	# b
7	9:5	7:4	36:35	down	48.8	7 7 b #
11	4:3	11:8	33:32	up	53.3	↑ ↓ # b
13	8:5	13:8	65:64	up	26.8	13 ε1 # 13 b
17	25:24	17:16	51:50	up	34.3	17 17 # 17 b
19	6:5	19:16	96:95	down	18.1	19 61 19 b 61 #
23	45:32	23:16	46:45	up	38.1	23 ε2 23 # 23 b
29	9:5	29:16	145:144	up	11.9	29 6z 29 # 29 b
31	15:8	31:16	31:30	up	56.8	31 1ε 31 # 31 b

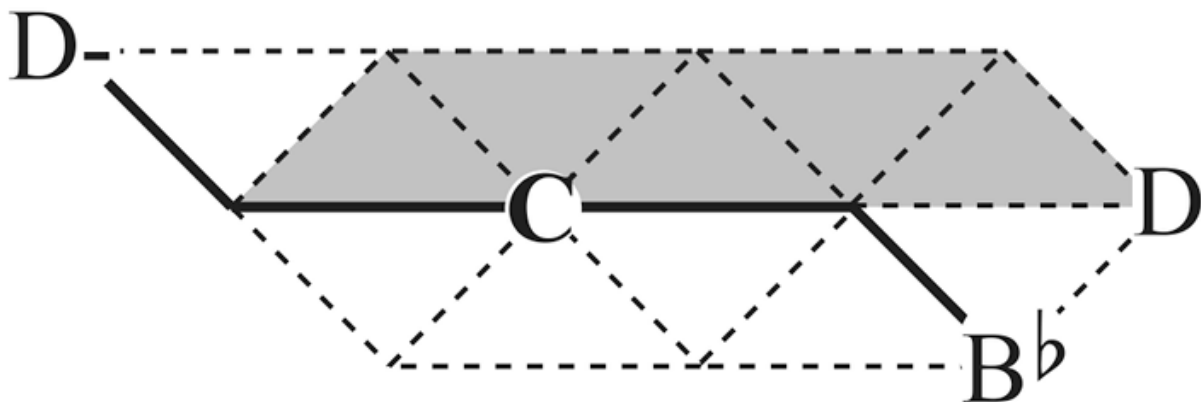
**Example 5.** The overtone and undertone series of C notated using Johnston's method

etc.

(8<sup>vb</sup>)

(8<sup>va</sup>)

**Example 6.** A *Tonnetz* with the syntonic diatonic highlighted in grey. The solid lines connect the two 5-limit pitches that may be inflected to produce a tonal or tonal seventh against the C. This makes clear why the tonal seventh of C is notated as lowered by a syntonic comma in addition to an inverse 7 sign. It is because the pitch is tuned relative to the D- (10:9), which is a syntonic comma lower than the D that appears in the diatonic gamut.



**Example 7.** Johnston, String Quartet No. 9/I, m. 109. A comma pump progression shown with Roman Numerals and a Tonnetz. The lines connecting circled pitches show the tuning path that leads to C lowered by a syntonic comma.

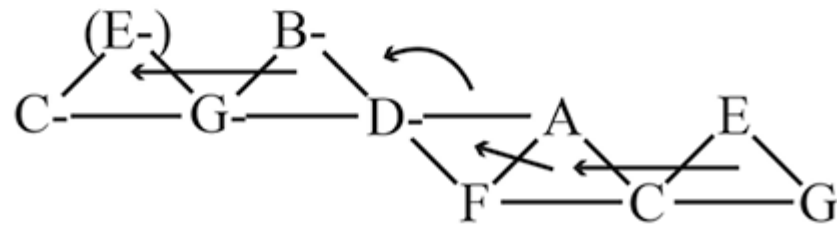
Violin I

Violin II

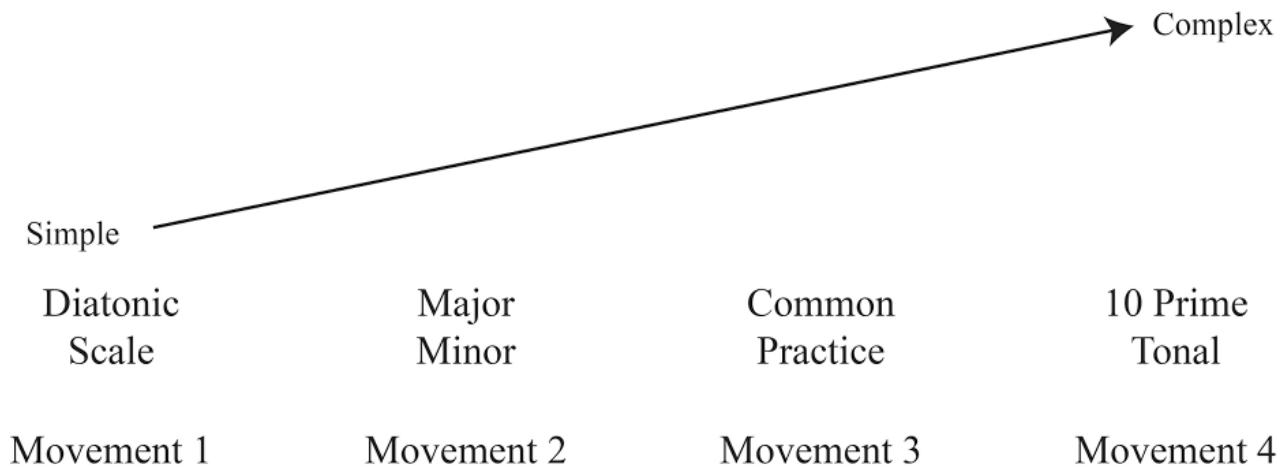
Viola

Violoncello

C+: I IV -ii -V (-I)



**Example 8.** The overall just-intonation form of String Quartet No. 9



**Example 9.** Johnston, String Quartet No. 9/I, RH A. The overlapping metric scheme of the body of the first movement. Notice that all the pitches of the C diatonic scale are present.

[A] ♩ = 48

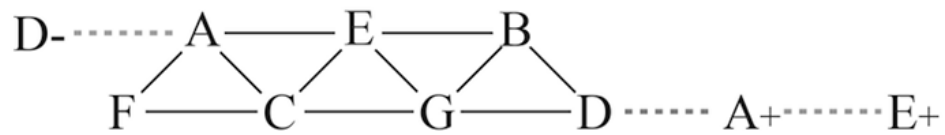
Violin I  $\frac{10}{16}$

Violin II  $\frac{12}{16}$

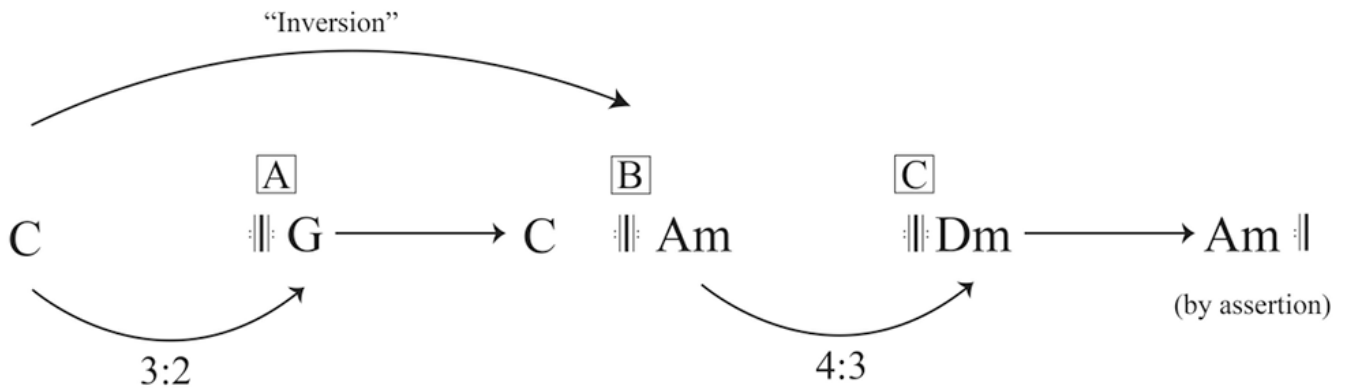
Viola  $\frac{9}{16}$

Violoncello  $\frac{2}{8}$

**Example 10.** Johnston, String Quartet No. 9/I, mm. 11–14. Asserted 3:2 relations are shown on a subset of the 5-limit Tonnetz and on the score with dotted lines.



**Example 11.** The tonal structure underlying the second movement with rehearsal marks and repeat signs



Example 12. Johnston, Quartet No. 9/II, mm. 1–14

**Fast, elated** ♩=288

Violin I  
Violin II  
Viola  
Violoncello

Example 13. The scale seen in Ex. 12, the overtone scale of C (Co17), in ratios and cents with additional marking of the 4:5:6 triads found in the scale

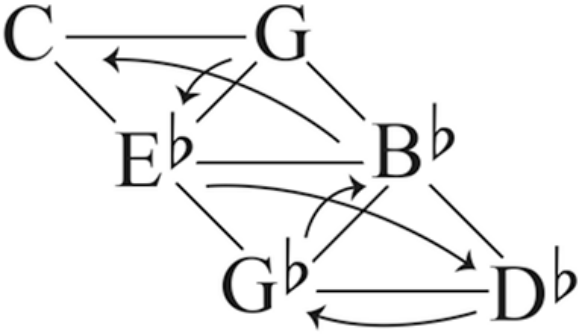
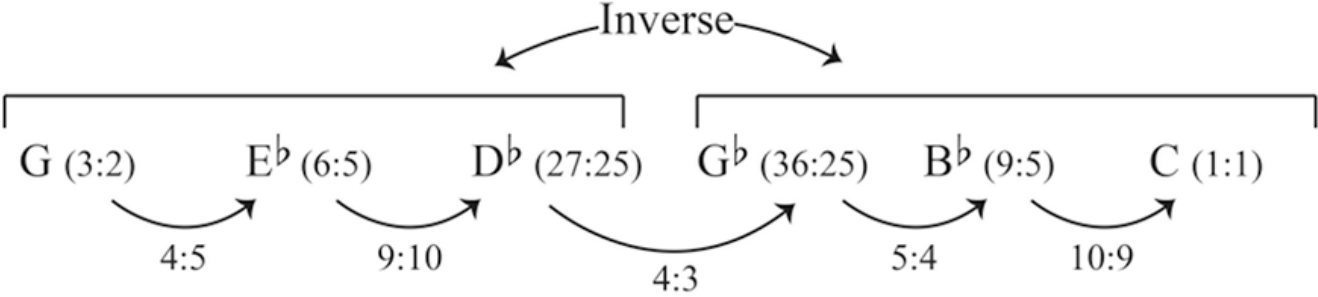
5-limit Triads	G major:									
	C major:									
C o17:	C	C <sup>♯</sup> (17:16)	D (9:8)	E (5:4)	F <sup>♯</sup> (11:8)	G (3:2)	A <sup>b</sup> (13:8)	B <sup>b</sup> (7:4)	B (15:8)	C (2:1)
Cents:	0	105	204	386	551	702	841	969	1088	1200
Interval Sizes:		17:16	18:17	10:9	11:10	12:11	13:12	14:13	15:14	16:15
		105	99	182	165	151	139	128	119	112
		9:8								
		204								



**Example 14.** Johnston, String Quartet No. 9/II, mm. 15–28. Local tonal centers are indicated beneath the score with ratios in reference to C as 1:1.

The image displays a musical score for a string quartet, consisting of four staves: Violin I, Violin II, Viola, and Violoncello. The music is written in 7/8 time and marked with a forte (*ff*) dynamic. The score is divided into two systems, each containing six measures. The first system (measures 15-20) features local tonal centers of G (3:2), E<sup>b</sup> (6:5), and D<sup>b</sup> (27:25). The second system (measures 22-28) features local tonal centers of G<sup>b</sup> (36:25), B<sup>b</sup> (9:5), and C (1:1). The notation includes various accidentals, slurs, and dynamic markings. The ratios are placed below the corresponding measures in the first system.

**Example 15.** The modulatory scheme of the B section of the second movement along with this motion on a *Tonnetz*



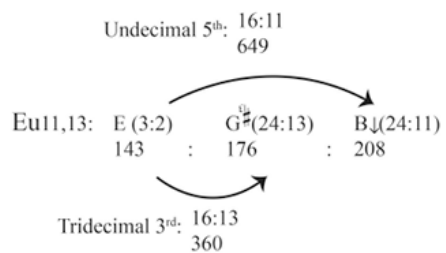
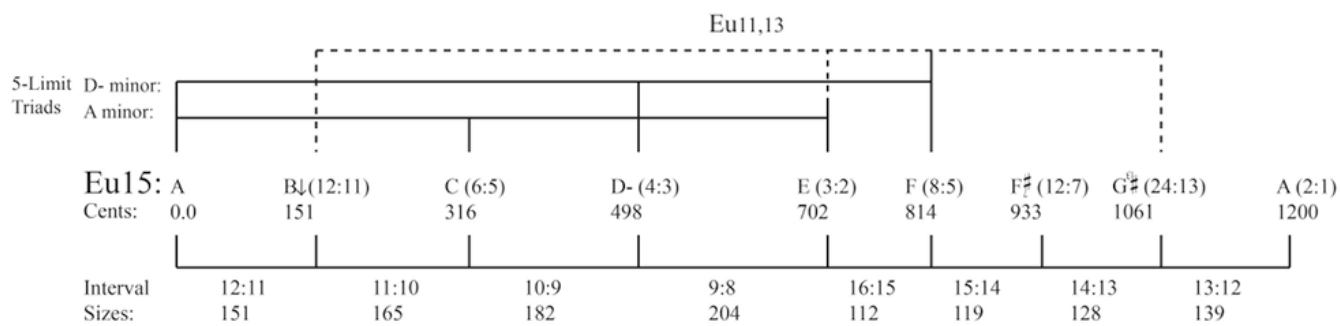
Example 16. Johnston, String Quartet No. 9/II, mm. 29–42

Musical score for measures 29-33. The score is for Violin I, Violin II, Viola, and Violoncello. The key signature has one sharp (F#) and the time signature is 7/8. The music is marked *ff* (fortissimo). The Violoncello part includes a *pizz.* (pizzicato) instruction. The measures show a complex rhythmic texture with various note values and rests.

Musical score for measures 34-37. The score continues for Violin I, Violin II, Viola, and Violoncello. The key signature and time signature remain the same. The music is marked *ff*. The measures feature intricate rhythmic patterns and melodic lines across all instruments.

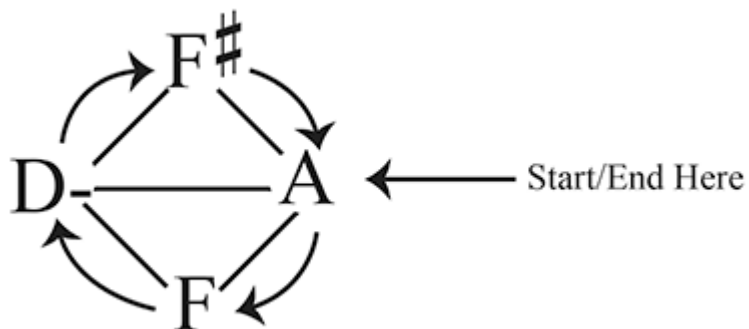
Musical score for measures 38-42. The score continues for Violin I, Violin II, Viola, and Violoncello. The key signature and time signature remain the same. The music is marked *ff*. The measures include a section marked "at frog" for the Violin I, II, and Viola parts, indicating a specific performance technique. The Violoncello part continues with its *ff* marking.

**Example 17.** The Eu15 scale with the structure of the “dominant” triad, Eu11,13, analyzed



**Example 18.** Johnston, String Quartet No. 9/II, mm. 43–56 with local tonics indicated beneath the score with their ratios as figured against C as 1:1. These modulations are also shown on a subset of the 5-limit Tonnetz.

The image displays a musical score for a string quartet, specifically measures 43 through 56. The score is arranged in four staves: Violin I, Violin II, Viola, and Violoncello. The key signature is one flat (B-flat major/D minor). The time signature is 7/8. The score is marked with dynamics such as *ff* and *f*. The Violoncello part includes a *pizz.* (pizzicato) instruction. The score is divided into four systems, each corresponding to a different local tonic indicated by a label below the staff: Au (5:3) at measure 43, F (4:3) at measure 47, D-u (10:9) at measure 49, and F#u (25:18) at measure 52. The notation includes various rhythmic values, accidentals, and articulation marks. The labels for the tonics are placed below the first two systems, and the label for the final tonic is placed below the fourth system.



**Example 19.** Johnston, String Quartet No. 9/III, mm. 1–15 with Roman Numeral analysis integrating Johnston’s syntonic comma notation

**Slow, expressive** ♩ = 66

Violin I *mp*

Violin II *mp*

Viola *mp*

Violoncello *mp*

F: I vi V I IV<sup>7</sup> I V I iii<sup>7</sup> vi<sup>7</sup> -ii<sup>7</sup> -V<sup>7</sup> -I [-iv<sup>7</sup><sub>3</sub> -V<sub>3</sub>

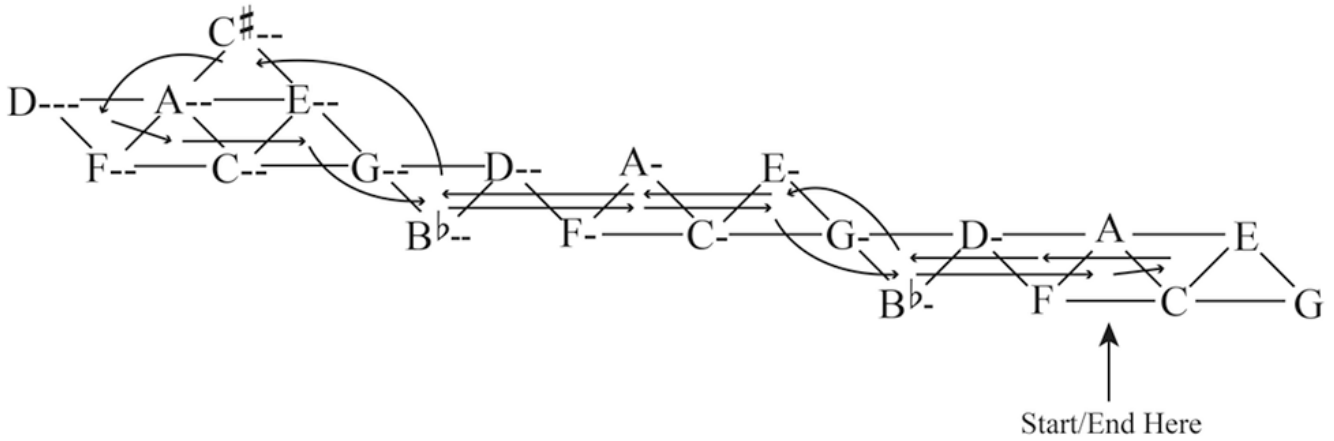
HC

8

7] --vi --I <sup>6</sup> --V--ii<sup>6</sup> -I <sup>6</sup> -V -ii<sup>6</sup> { I I<sup>6</sup> ii<sup>7</sup> V<sup>7</sup> I V <sup>7</sup> I

C: (V) PAC HC

**Example 20.** The triadic motion of the first fourteen measures of the third movement represented on a 5-limit *Tonnetz*



**Example 21.** Johnston, String Quartet No. 9/III, mm. 50–51 with the ratio of higher prime passing notes shown beneath the score along with the inverse relationship between the major tonic and minor submediant harmonies

$$\begin{array}{c}
 13:12 \quad 40:39 \\
 7:6 \quad 40:21 \\
 7:4 \quad 4:3 \quad 13:8 \quad 4:7 \quad 5:3 \quad 8:13 \\
 \text{F: I} \quad \text{vi} \\
 \text{Inversion}
 \end{array}$$

**Example 22.** Johnston, String Quartet No. 9/III, mm. 57–63 with Roman Numeral analysis integrating Johnston’s syntonic comma notation

Violin I  
Violin II  
Viola  
Violoncello

F: [V<sup>7</sup>] --vi --I 6 --V --ii<sup>6</sup>

-I 6 -V -ii<sup>6</sup>

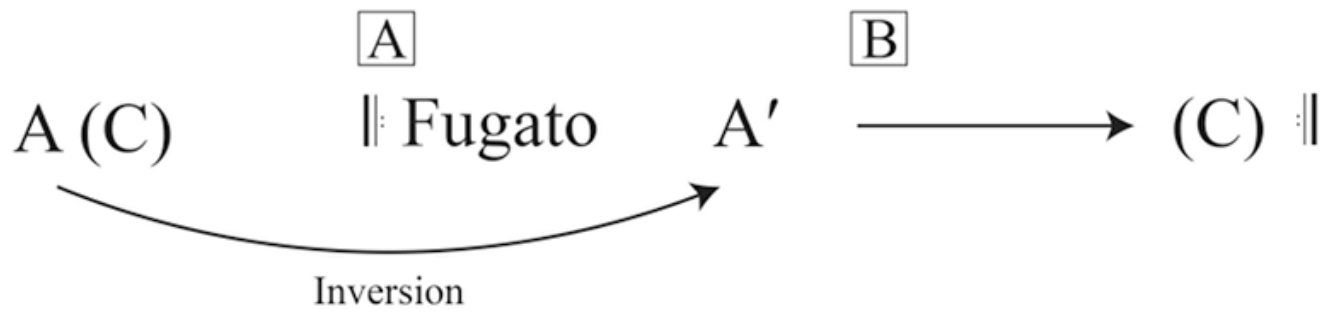
*rall.*

*ff*  
*ff*  
*ff*  
*ff*

I vi IV V(♯<sup>7</sup>) I



**Example 23.** The form of String Quartet No. 9/IV with rehearsal marks and tonality in brackets



**Example 24.** Johnston, String Quartet No. 9/IV, mm. 1–17 with scale resources indicated beneath the score and the modulation to the dominant analyzed using Roman Numerals

Violin I  
Violin II  
Viola  
Violoncello

*ff*

Co13 (Go7) Cu13

5

Co13

8

Co13

10

Co23 Co31

12

Co31

14

Gu31

G: V( $\frac{3}{4}$ ) (V) (V/ii)  $\frac{5}{4}$

16

7) Do17

I

pizz.

**Example 25.** Johnston, String Quartet No. 9/IV, mm. 41–52 with scale resources indicated beneath the score and the modulation to the tonic analyzed using Roman Numerals

Gu23

Gu31

C: V( $\frac{4}{4}$ ) (V/ii)  $\frac{3}{4}$   $\frac{7}{4}$  Go17 I