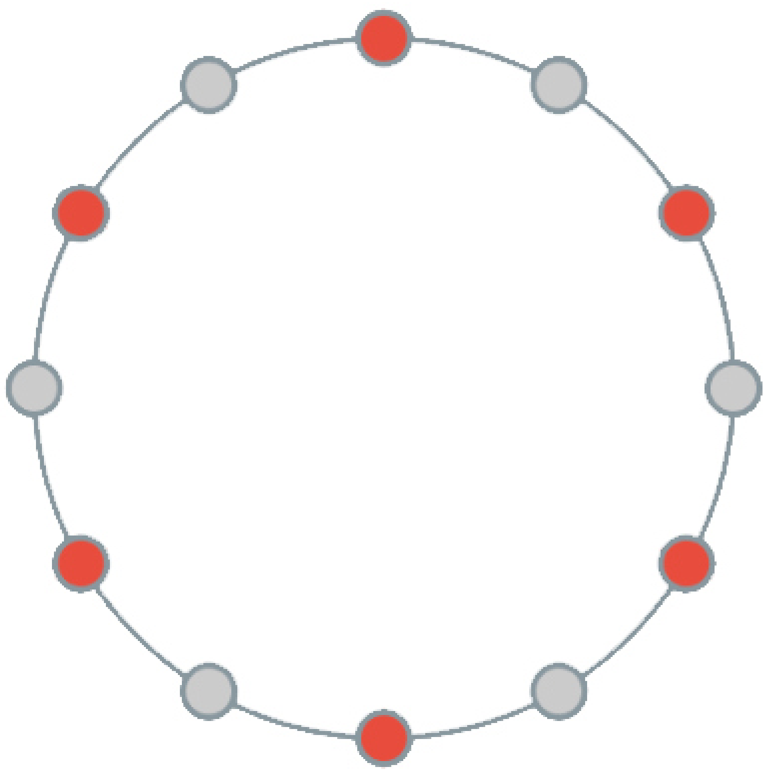
A JOURNAL OF THE SOCIETY FOR MUSIC THEORY

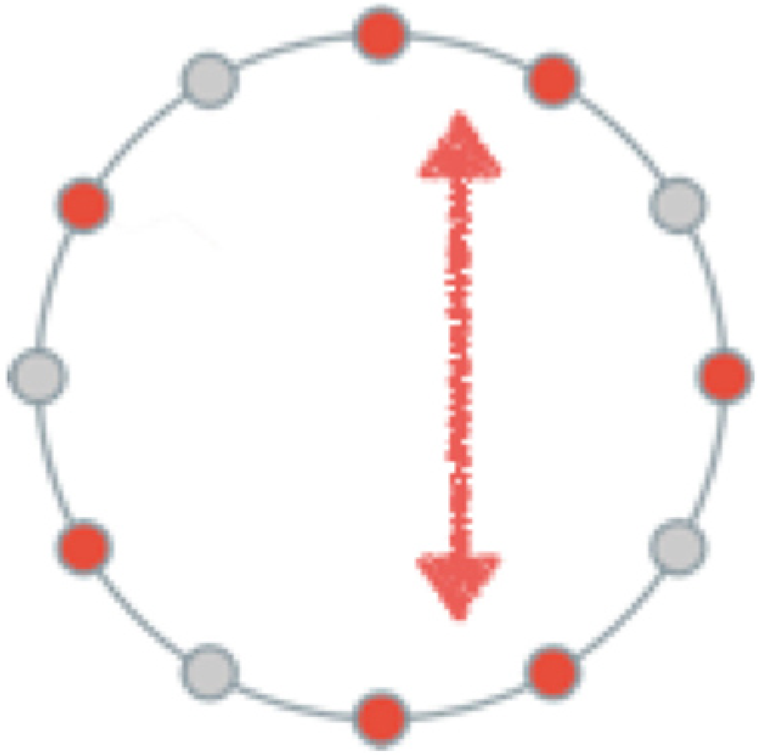
MTO 25.2 Examples: Plotkin, Chord Proximity

(Note: audio, video, and other interactive examples are only available online)
http://mtosmt.org/issues/mto.19.25.2/mto.19.25.2.plotkin.html

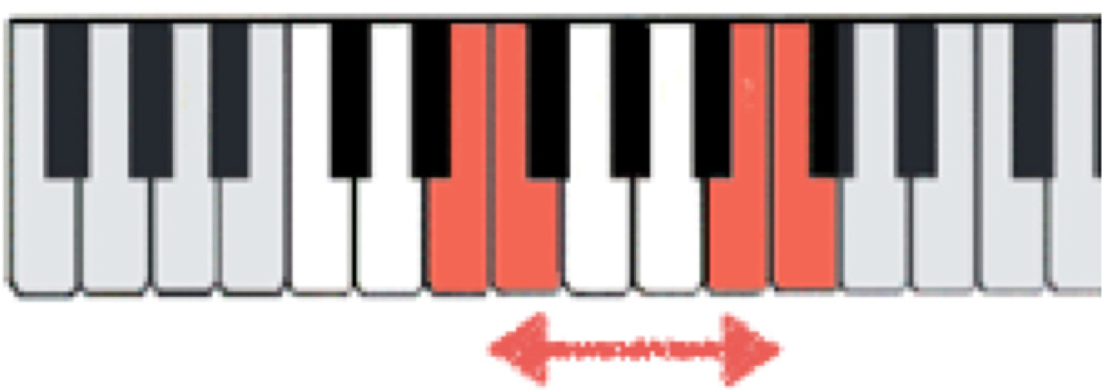
Example 1. A totally even distribution of 6→12



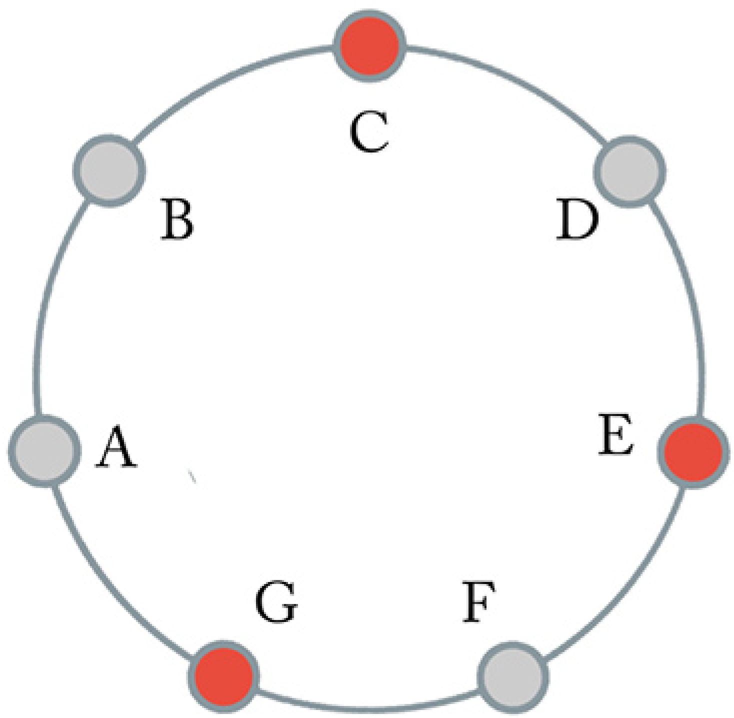
Example 2a. The pairs of adjacent red dots are as far apart as possible, as indicated by the red arrow



Example 2b. On the keyboard, the pairs of adjacent white keys are as far apart as possible (for a white-note collection)



Example 3. A C+ triad as a maximally even distribution of three notes in a 7-note collection



(Video) Example 4. 7→12 and 3→7 combine to visually portray a 2nd-order maximally even set

(Video) Example 5. Seven pentatonic collections as part of a 5→12 FiPS configuration

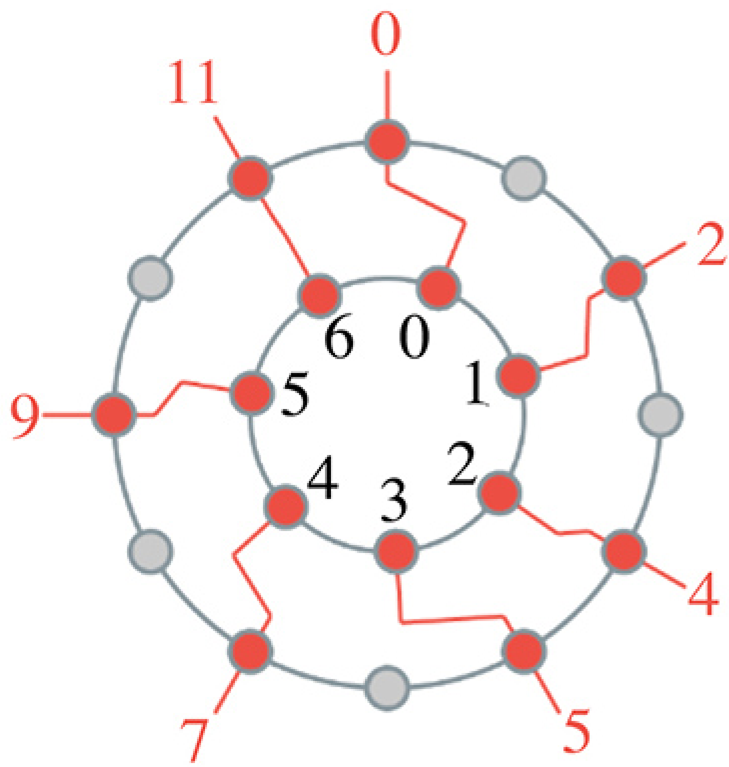
(Video) Example 6. Seven diatonic collections as part of a 7→12 FiPS configuration

(Video) Example 7. Three octatonic collections as part of an 8→12 FiPS configuration

(Video) Example 8. Seven implicitly diatonic chords as part of a 3→7 FiPS configuration

(Video) Example 9. Seven explicitly diatonic chords as part of a 3→7→12 FiPS configuration

Example 10. A 7→12 maximally even distribution, with connections between indices explicitly traced



Example 11. (a) A calculation of $J^{m_2}_{7,3}$

in $J^{m_1,m_2}_{12,7,3}$, where $m_2=0$; (b) A calculation of $J^{m_2}_{7,3}$ in $J^{m_1,m_2}_{12,7,3}$, where $m_2=1$

. There is an increase of 1/3 in each value before the floor function is applied

(a)	(b)
$J^0_{7,3}(0) = \left\lfloor 0\tfrac{0}{3} \right\rfloor = 0$	$J^1_{7,3}(0) = \left\lfloor 0\tfrac{1}{3} \right\rfloor = 0$
$J^0_{7,3}(1) = \left\lfloor 2\tfrac{1}{3} \right\rfloor = 2$	$J^1_{7,3}(1) = \left\lfloor 2\tfrac{2}{3} \right\rfloor = 2$
$J^0_{7,3}(2) = \left\lfloor 4\tfrac{2}{3} \right\rfloor = 4$	$J^1_{7,3}(2) = \left\lfloor 4\tfrac{3}{3} \right\rfloor = 5$

Example 12. There is an equal distance of 30° (30° = 360/12) between points on the outer ring. Each remainder corresponds to the distance between the beam of the beacon and the distance it must travel counter-clockwise to reach an evenly-spaced hole on the outer ring.

$J^{5}_{12,7}(0) = \lfloor 0\,\tfrac{5}{7} \rfloor$, remainder of $5/7$

$J^{5}_{12,7}(1) = \lfloor 2\,\tfrac{3}{7} \rfloor$, remainder of $3/7$

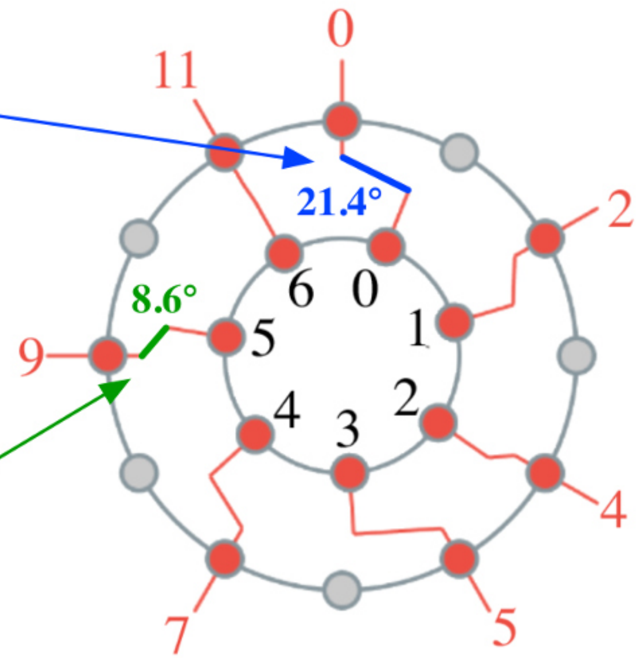
$J^{5}_{12,7}(2) = \lfloor 4\,\tfrac{1}{7} \rfloor$, remainder of $1/7$

$J^{5}_{12,7}(3) = \lfloor 5\,\tfrac{6}{7} \rfloor$, remainder of $6/7$

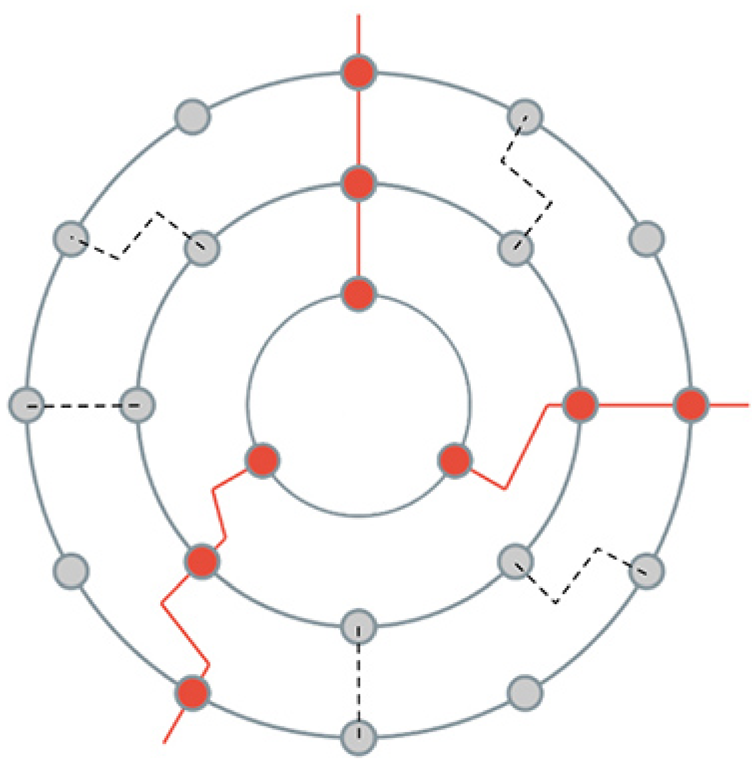
$J^{5}_{12,7}(4) = \lfloor 7\,\tfrac{4}{7} \rfloor$, remainder of $4/7$

$J^{5}_{12,7}(5) = \lfloor 9\,\tfrac{2}{7} \rfloor$, remainder of $2/7$

$J^{5}_{12,7}(6) = \lfloor 11\,\tfrac{0}{7} \rfloor$, remainder of $0/7$

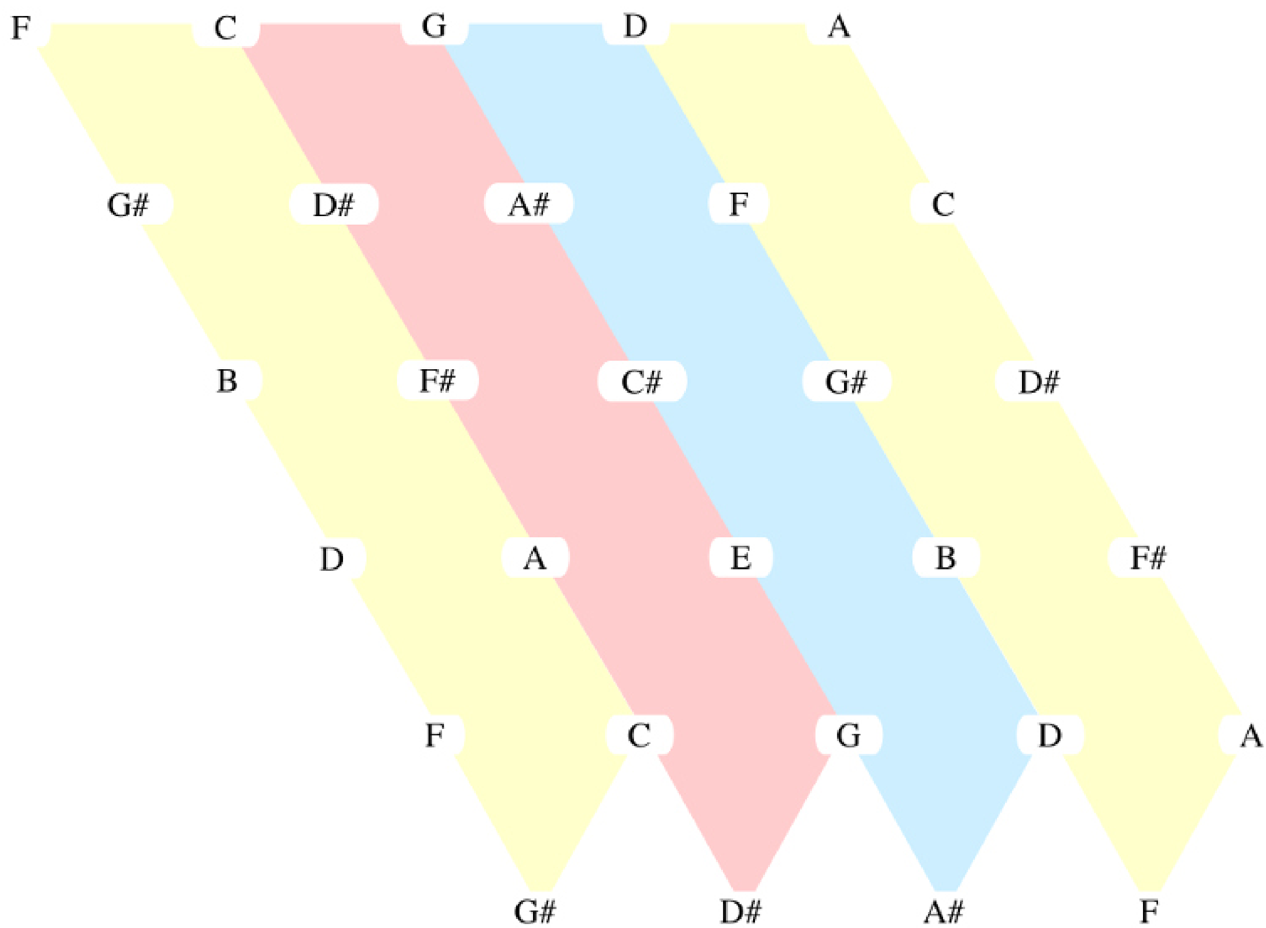


Example 13. A 3→8→12 configuration in which a C- triad is the result

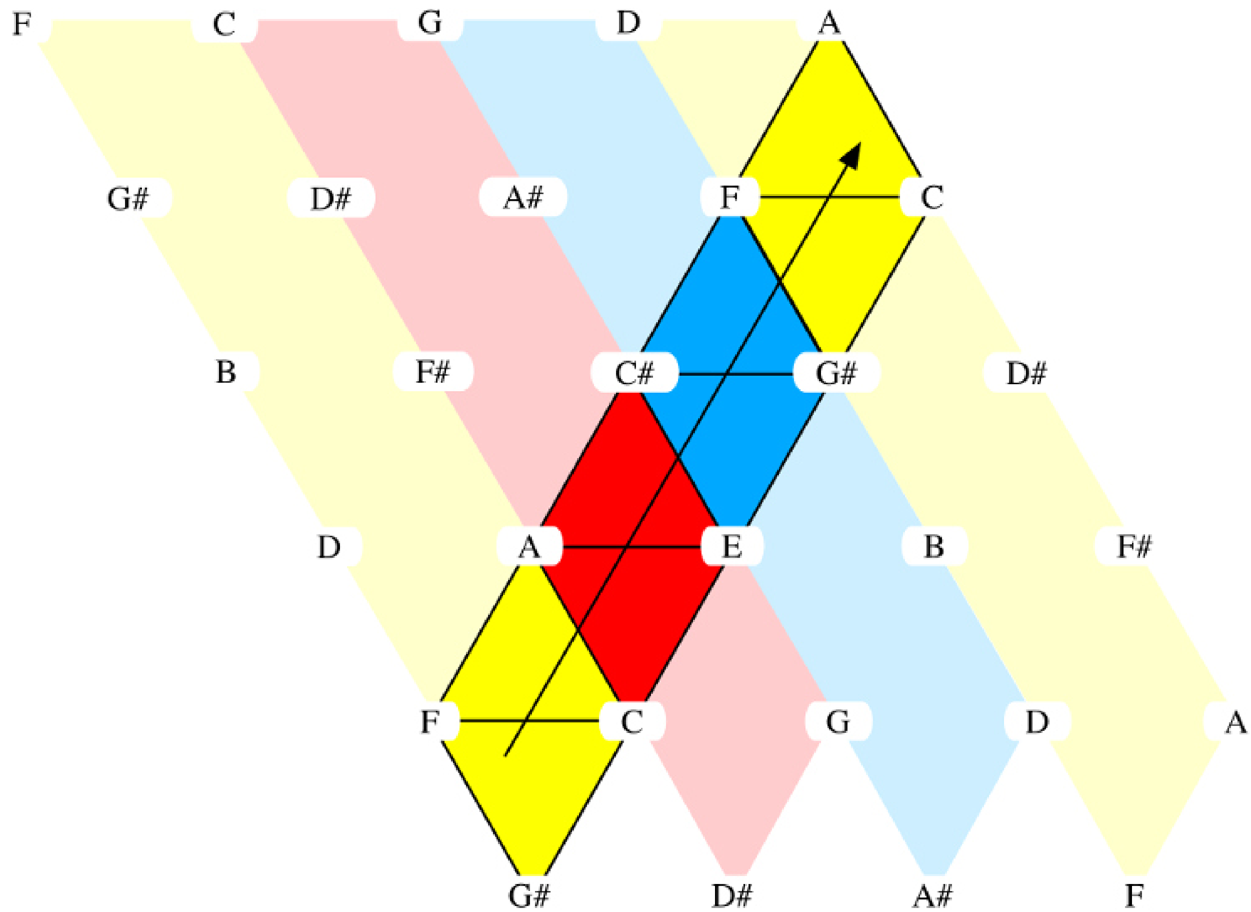


(Video) Example 14. A 3→8→12 configuration in which all members of the 01-octatonic are realized

Example 15. The neo-Riemannian *Tonnetz*, with highlighted octatonic corridors

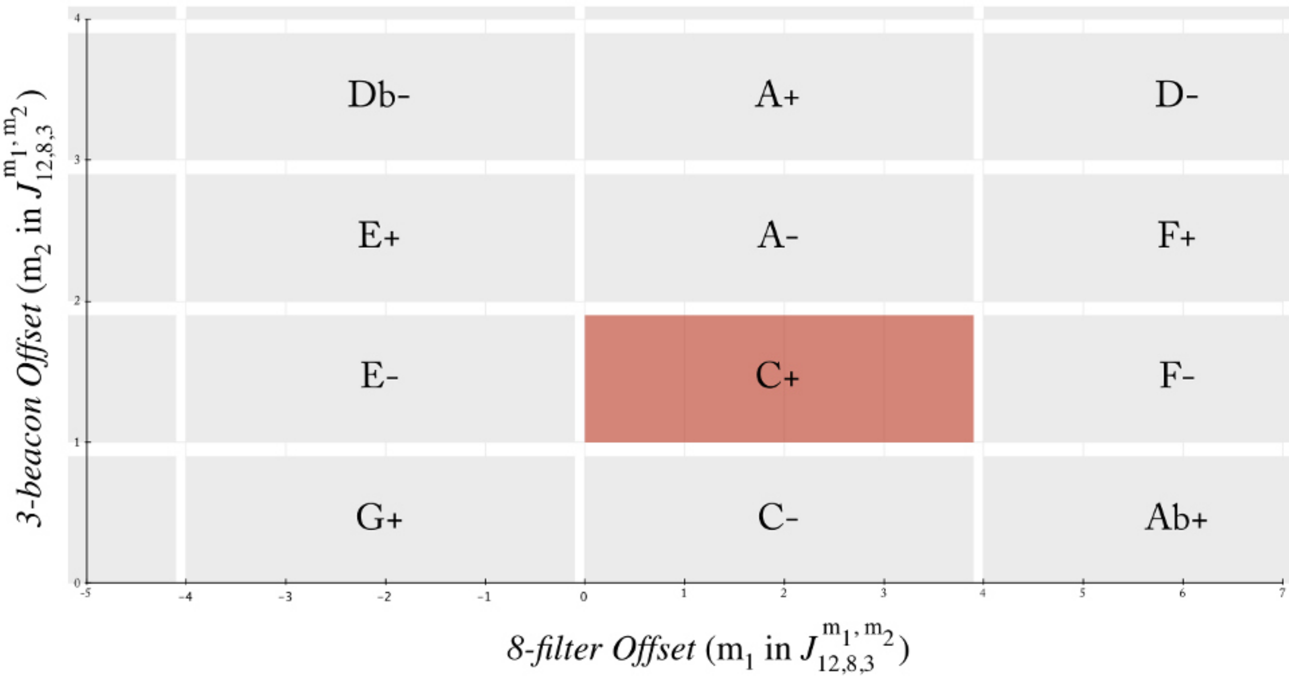


Example 16. The neo-Riemannian *Tonnetz* with highlighted octatonic corridors, and a *PL* cycle following a path *within* and then *out into* each corridor

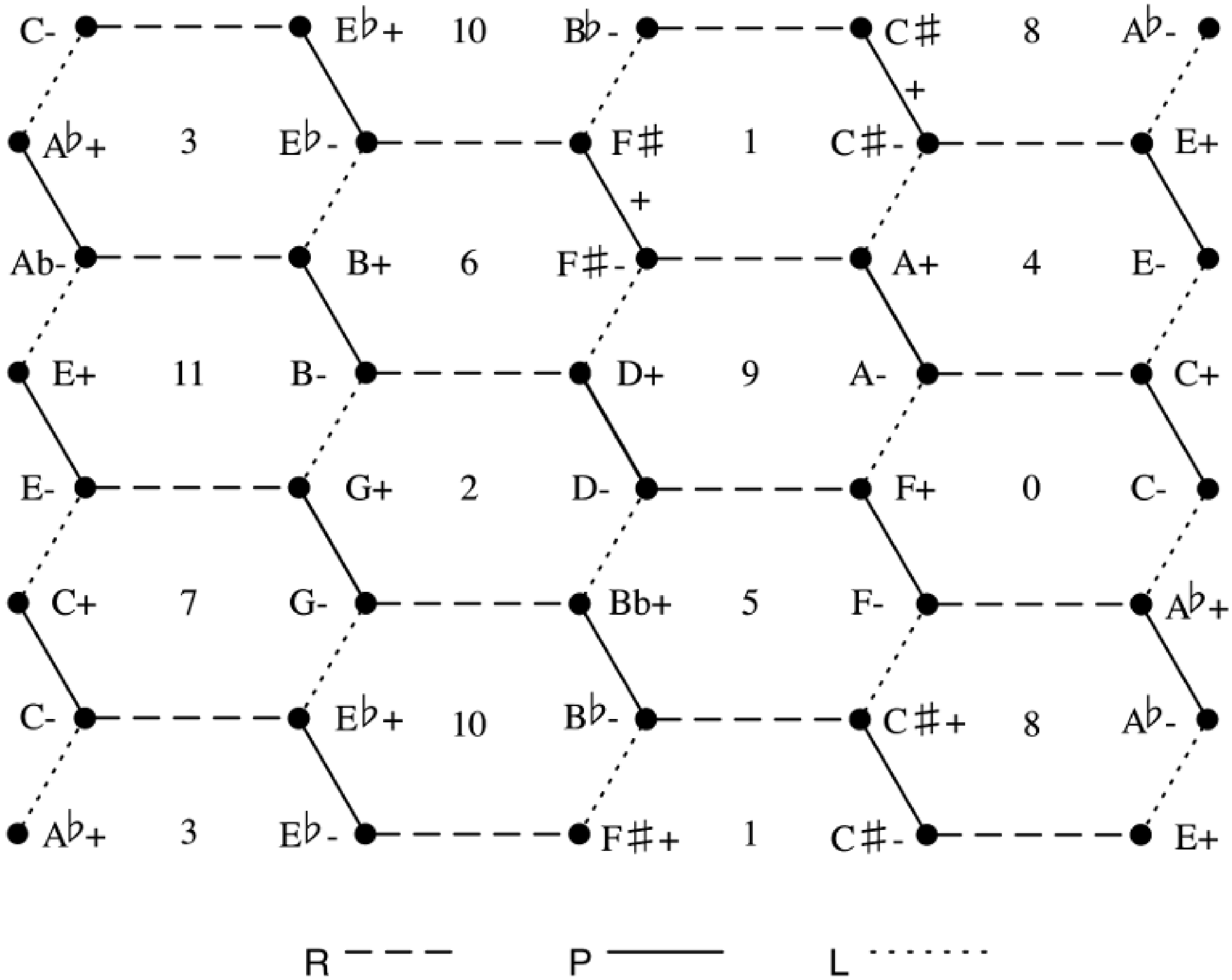


(Video) Example 17. A 3→8→12 configuration in which a *PL* cycle is realized, in the manner of Example 16

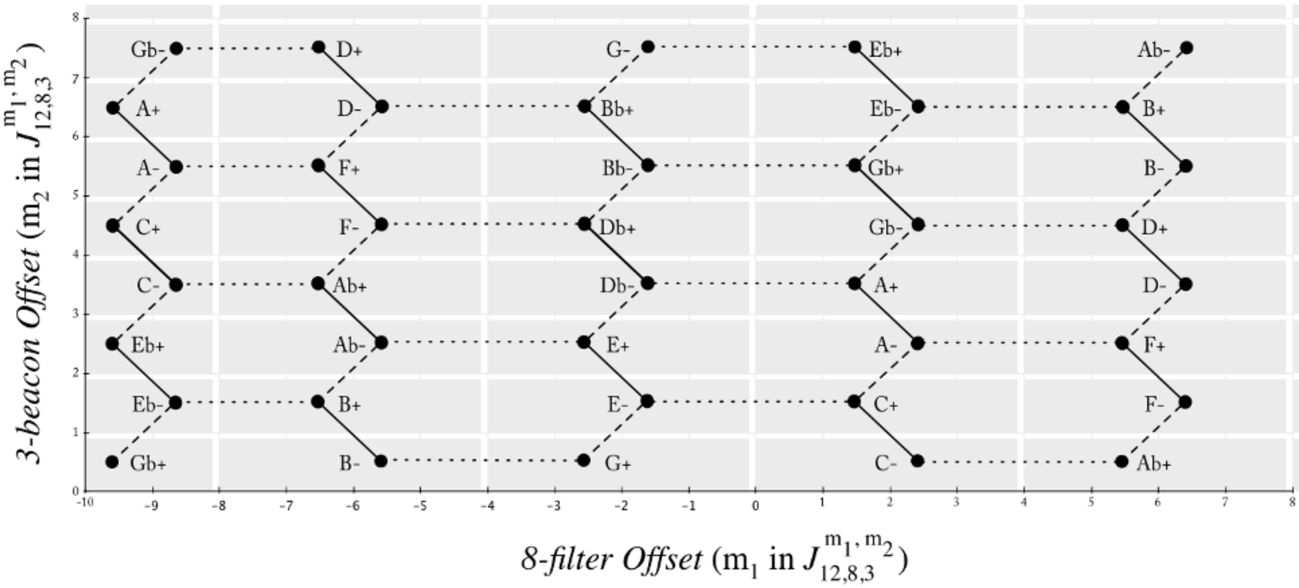
Example 18. A 3→8→12 configuration space



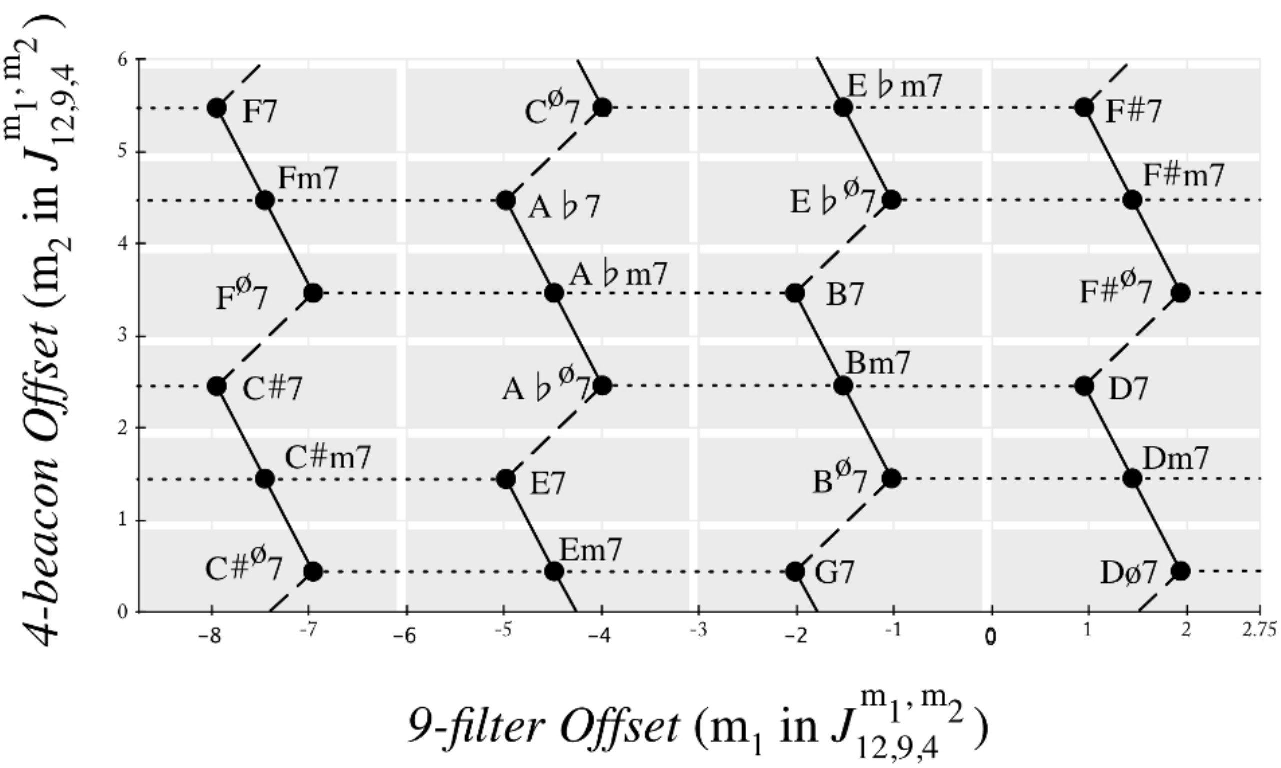
Example 19. Douthett and Steinbach's (1998) Chicken-Wire Torus



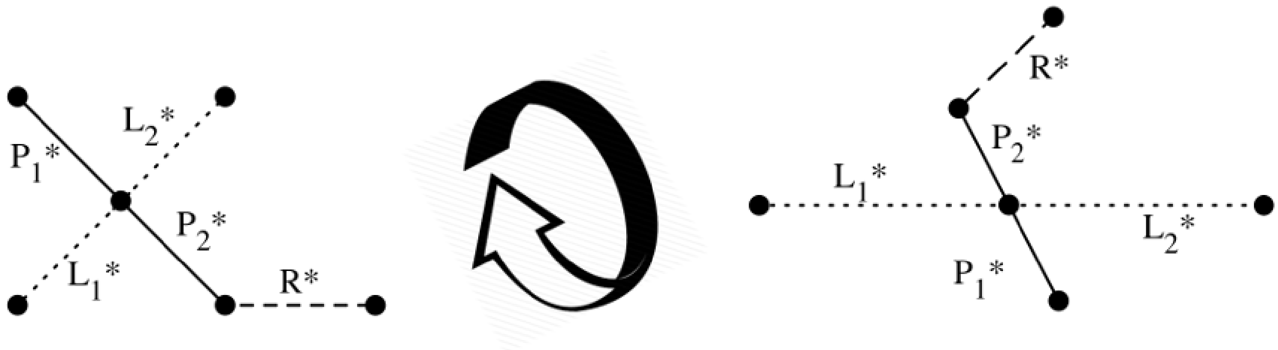
Example 20. A portion of a 3→8→12 configuration space, with a matching portion of the Chicken-Wire Torus overlaid to demonstrate the isomorphism



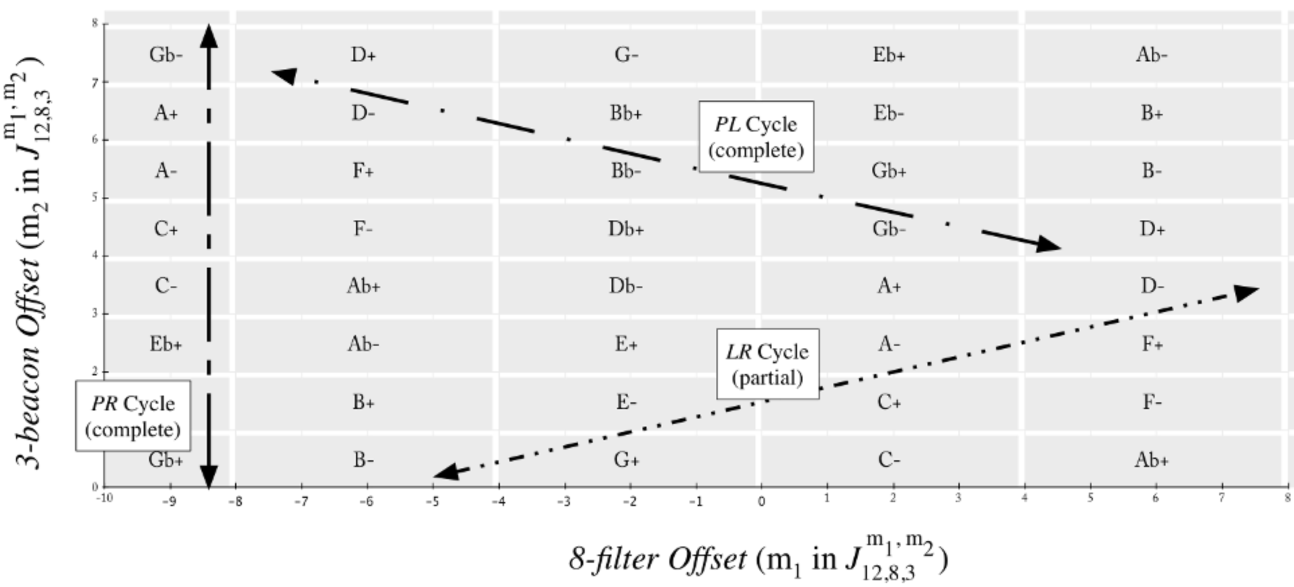
Example 21. A portion of a 4→9→12 configuration space, with a matching portion of the Towers Torus overlaid to demonstrate the isomorphism



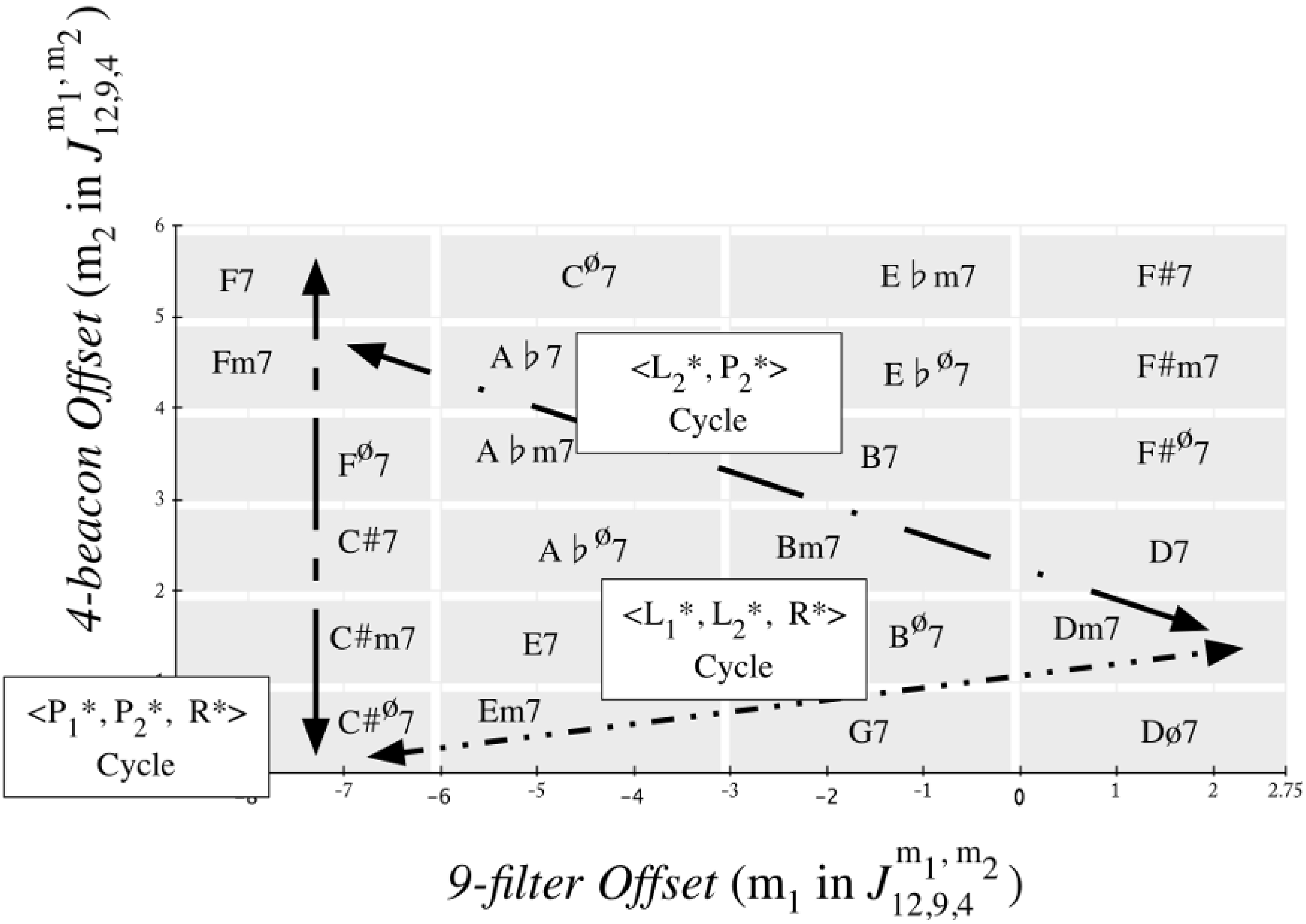
Example 22. On the left, the original key to Towers Torus from Douthett and Steinbach. On the right, a rotated and slightly stretched version of the key that reflects the altered positioning of the diagram when overlaid on the 4→9→12 configuration space



Example 23. A 3→8→12; configuration space with neo-Riemannian cycles identified



Example 24. A 4→9→12 configuration space with Towers Torus cycles identified

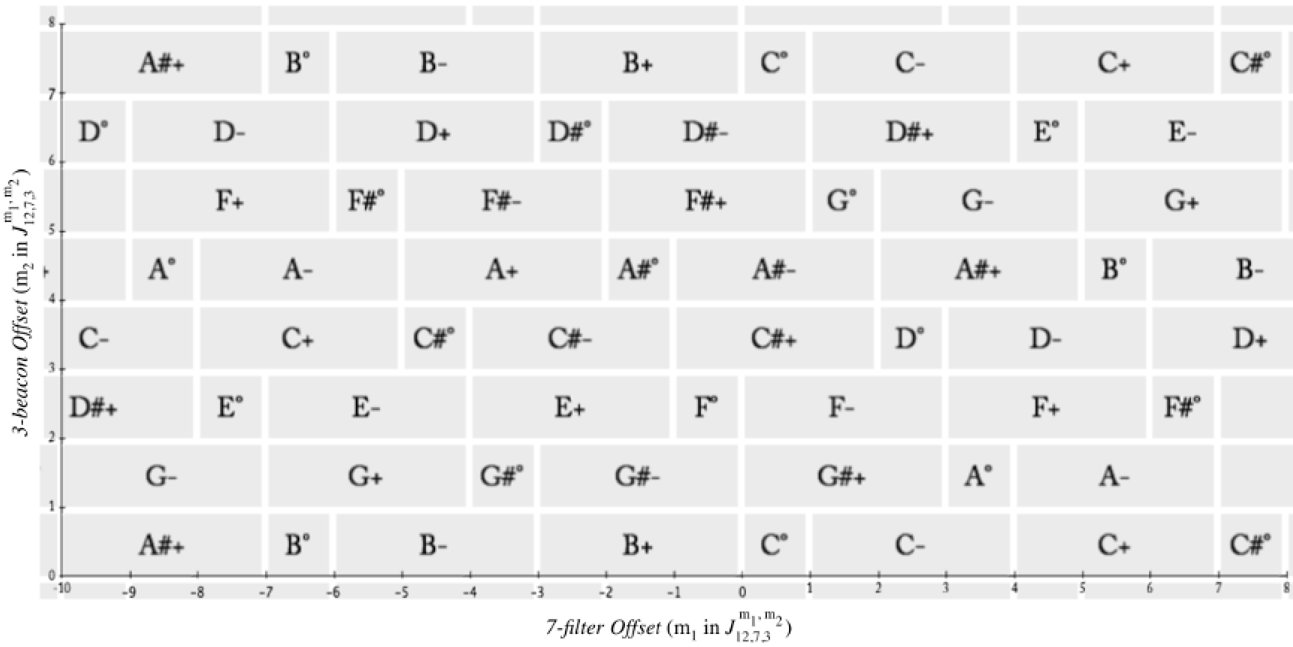


(Video) Example 25. The <P_1*, P_2*, R*> cycle identified in Example 24

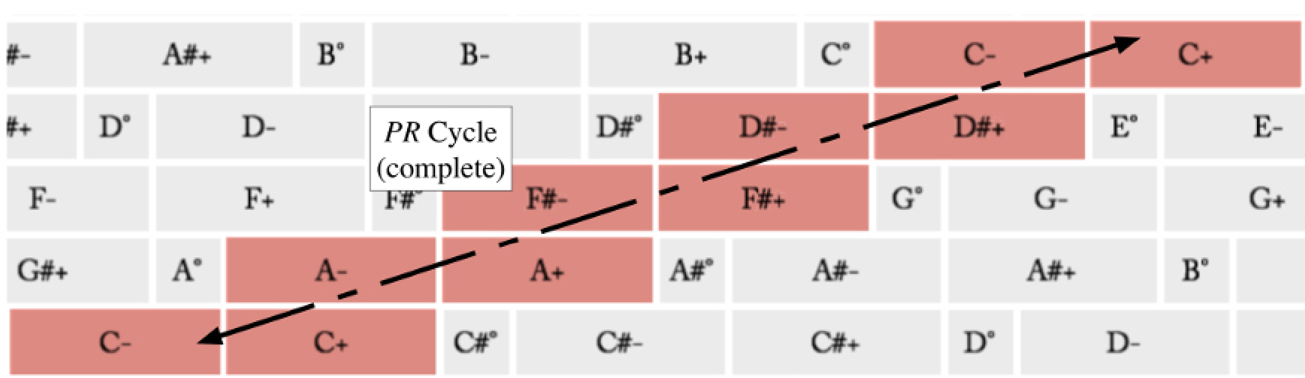
(Video) Example 26. The <P_2*, L_2*> cycle identified in Example 24

(Video) Example 27. A parsimonious cycle of diatonic trichords in C major

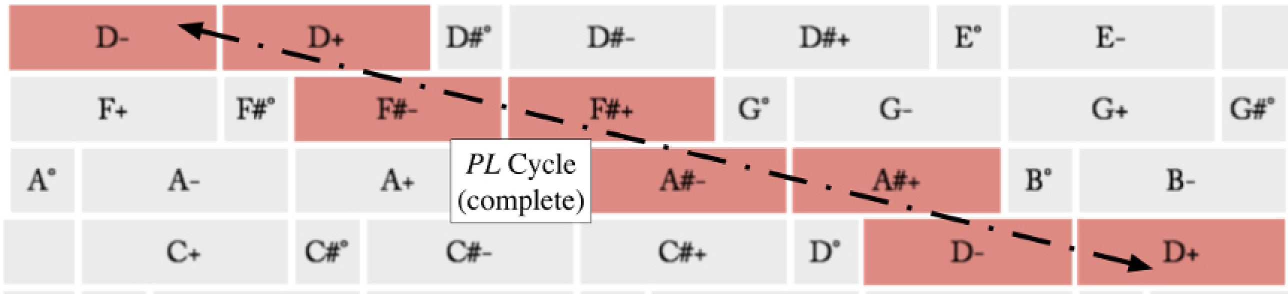
Example 28. A 3→7→12 configuration space



Example 29. A *PR* cycle in a 3→7→12 configuration space



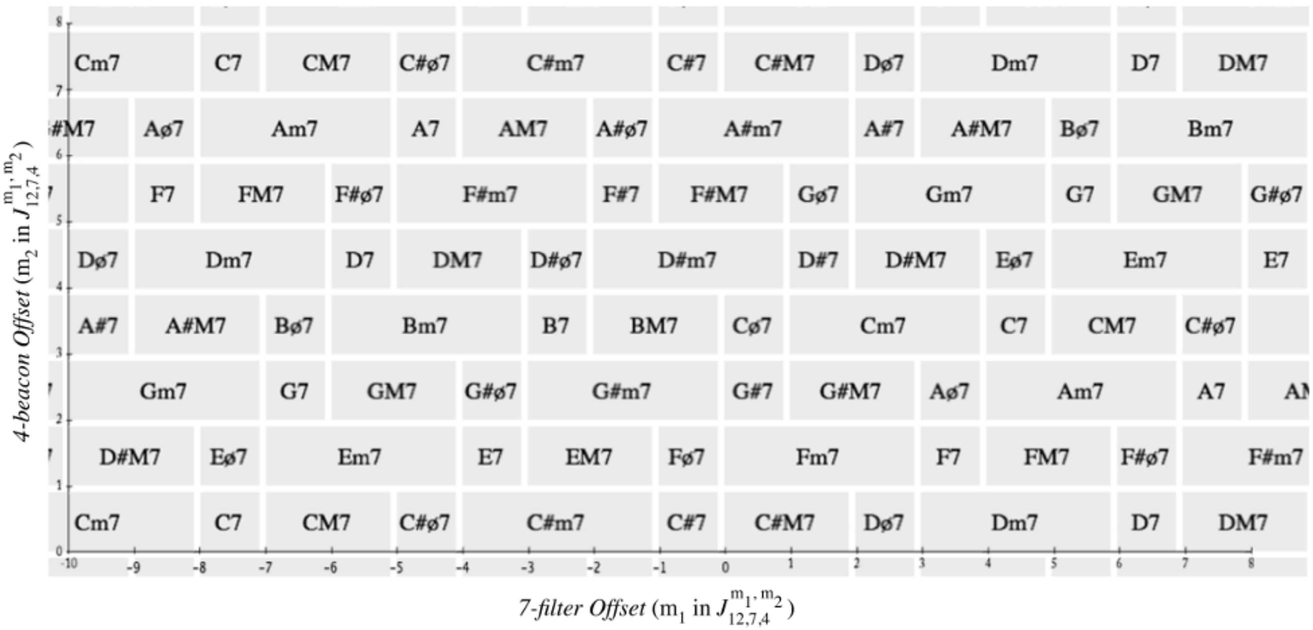
Example 30. A *PL* cycle in a 3→7→12 configuration space



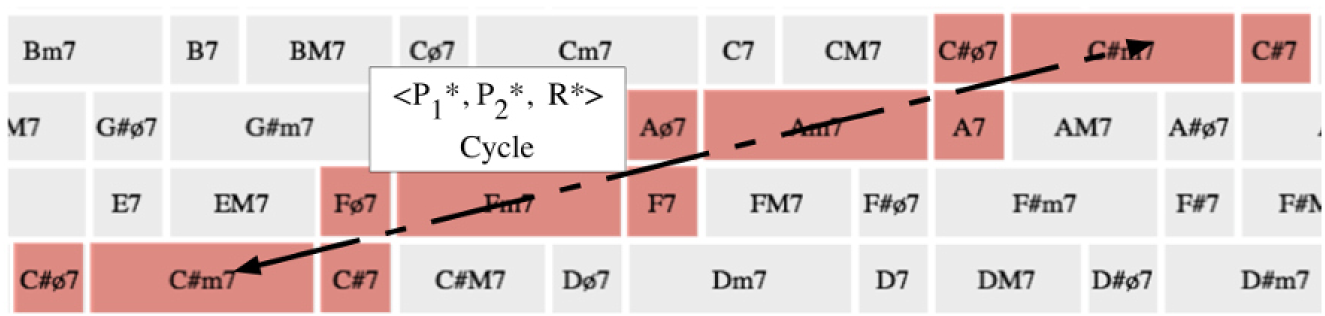
Example 31. An *LR* cycle in a 3→7→12 configuration space

Example 32. An *LR* cycle in a 3→5→12 configuration space. Note that the sus2 designations are arbitrary (in the same way one might choose the root of a fully-diminished seventh chord for a configuration space). These sus2 chords could easily be given as alternate-root sus4 chords, or relationships *between* these chords might be more apparent.

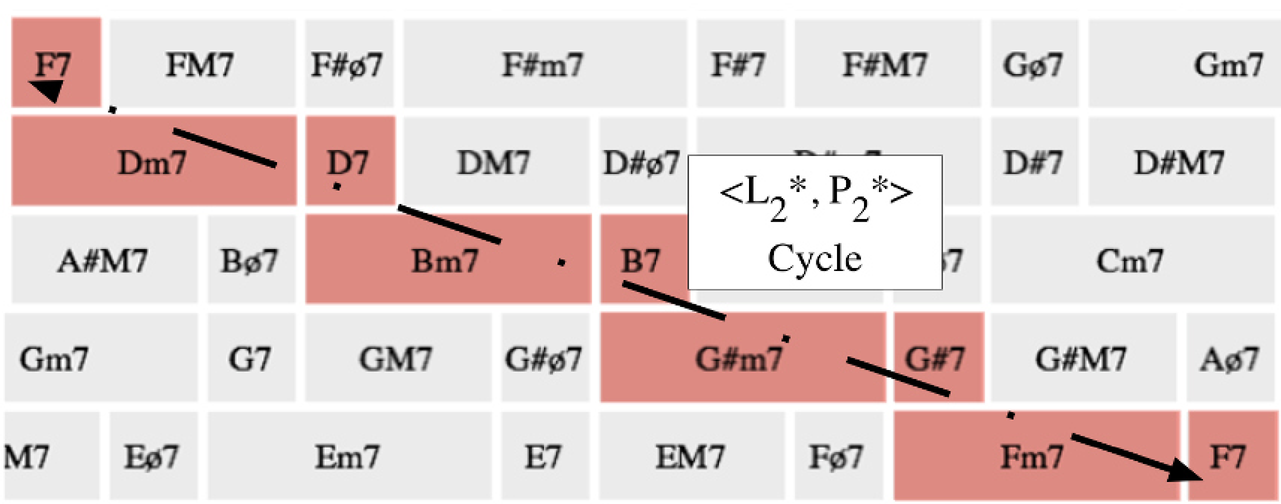
Example 33. A 4→7→12 configuration space



Example 34. A <*P₁**, *P₂**, *R**> cycle in a 4→7→12 configuration space



Example 35. An <*L₂**, *P₂**> cycle in a 4→7→12 configuration spac



Example 36. A cycle in a 4→7→12 configuration space that augments an <L_1*, L_2*, R*> cycle

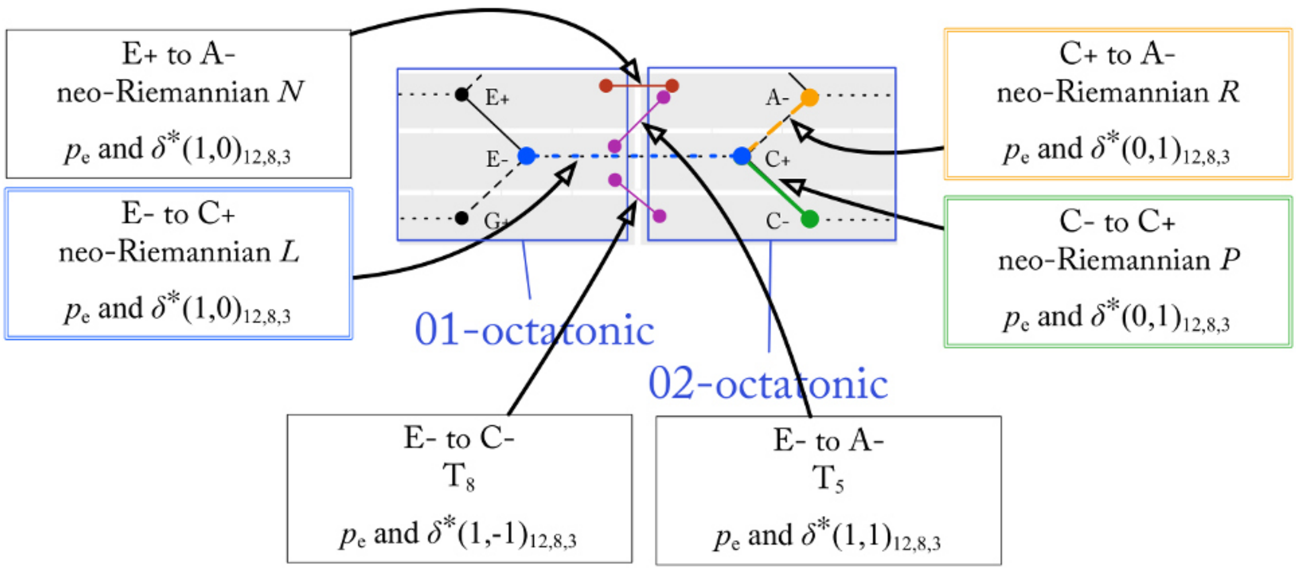
Example 37. δ* in various configuration spaces

Example 37: δ^* in various configuration spaces

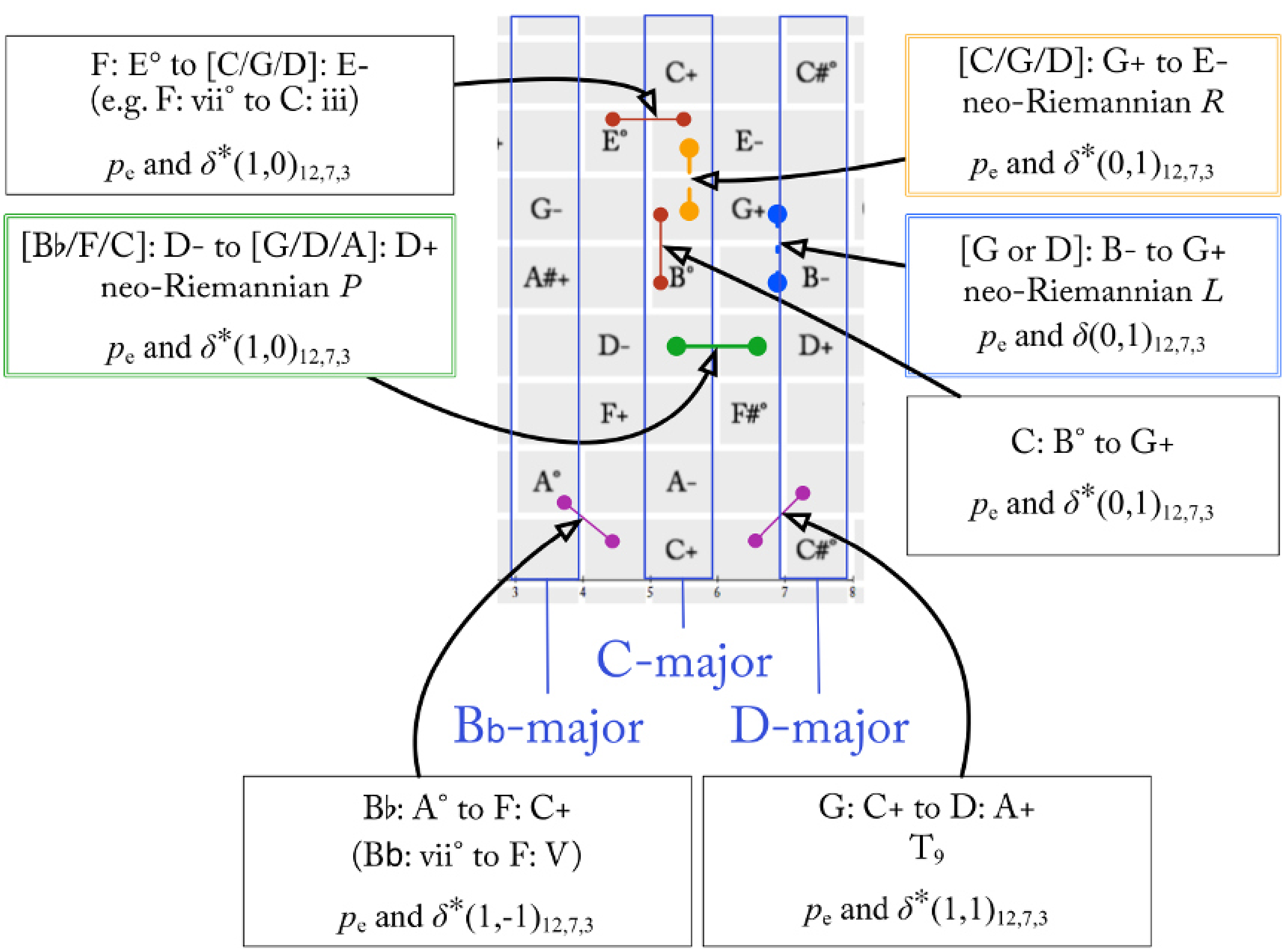
Set Type	Up $\delta^*(0,1)$	Down $\delta^*(0,-1)$	Right $\delta^*(1,0)$	Left $\delta^*(-1,0)$	Up-Right $\delta^*(1,1)$	Down-Left $\delta^*(-1,-1)$	Down-Right $\delta^*(1,-1)$	Up-Left $\delta^*(-1,1)$
3→8→12 (Fig. 23)								
Major Triad {0, 4, 7}	R {0, 4, **9**}	P {0, **3**, 7}	N {0, **5**, **8**}	L {**E**, 4, 7}	T_5 {0, **5**, **9**}	T_7 {**E**, **2**, 7}	T_8 {0, **3**, **8**}	T_4 {**E**, 4, **8**}
Minor Triad {0, 3, 7}	P {0, **4**, 7}	R {**E**, 3, 7}	L {0, 3, **8**}	N {**E**, **2**, 7}	T_5 {0, **5**, **8**}	T_7 {**T**, **2**, 7}	T_8 {**E**, 3, **8**}	T_4 {**E**, **4**, 7}
3→7→12 (Fig. 28)								
Major Triad {0, 4, 7}	R {0, 4, **9**}	N/A	T_1 + dim {**1**, 4, 7}	P {0, **3**, 7}	T_9 {**1**, 4, **9**}	T_3 {**T**, **3**, 7}	L {**E**, 4, 7}	T_9 + dim {0, **3**, **9**}
Minor Triad {0, 3, 7}	N/A	R {**E**, 3, 7}	P {0, **4**, 7}	T_0 + dim {0, 3, **6**}	T_9 {0, **4**, **9**}	T_3 {**T**, 3, **6**}	T_9 + dim {0, 3, **9**}	L {0, 3, **8**}
Diminished Triad {0, 3, 6}	T_8 + major {0, 3, **8**}	T_3 + minor {**T**, 3, 6}	T_0 + minor {0, 3, **7**}	T_{11} + major {**E**, 3, 6}	T_8 + major {0, 3, **8**}	T_3 + minor {**T**, 3, 6}	T_3 + major {**T**, 3, **7**}	T_8 + minor {**E**, 3, **8**}
4→9→12 (Fig. 21)								
Dom7 {0, 4, 7, T}	R* {**2**, 4, 7, T}	P_2* {0, **3**, 7, T}	T_7 + ø7 {**1**, **5**, 7, T}	L_2* {0, 4, 7, **9**}	T_7 + m7 {**2**, **5**, 7, T}	T_9 + ø7 {0, **3**, 7, **9**}	T_3 {**1**, **3**, 7, T}	T_9 {**1**, 4, 7, **9**}
m7 {0, 3, 7, T}	P_2* {0, **4**, 7, T}	P_1* {0, 3, **6**, T}	L_2* {**1**, 3, 7, T}	L_1* {0, 3, 7, **9**}	T_7 + ø7 {**1**, **5**, 7, T}	T_5 + Dom7 {0, 3, **5**, **9**}	T_3 {**1**, 3, **6**, T}	T_9 {0, **4**, 7, **9**}
ø7 {0, 3, 6, T}	P_1* {0, 3, **7**, T}	R* {0, 3, 6, **8**}	L_1* {**1**, 3, 6, T}	T_5 + Dom7 {0, 3, **5**, **9**}	T_3 + Dom7 {**1**, 3, **7**, T}	T_5 + m7 {0, 3, **5**, **8**}	T_3 {**1**, 3, 6, **9**}	T_9 {0, 3, **7**, **9**}
4→7→12 (Fig. 33)								
Dom7 {0, 4, 7, T}	R* {**2**, 4, 7, T}	L_2* {0, 4, 7, **9**}	T_0 + M7 {0, 4, 7, **E**}	P_2* {0, **3**, 7, T}	T_4 + m7 {**2**, 4, 7, **E**}	T_9 + ø7 {0, **3**, 7, **9**}	L_2* {0, 4, 7, **9**}	T_3 + M7 {**2**, **3**, 7, T}
m7 {0, 3, 7, T}	N/A	N/A	P_2* {0, **4**, 7, T}	P_1* {0, 3, **6**, T}	T_4 + ø7 {**2**, **4**, 7, T}	T_8 + Dom7 {0, 3, **6**, **8**}	T_9 {0, **4**, 7, **9**}	T_3 {**1**, 3, **6**, T}
ø7 {0, 3, 6, T}	L_1* {**1**, 3, 6, T}	T_8 + Dom7 {0, 3, 6, **8**}	P_1* {0, 3, **7**, T}	T_{11} + M7 {**E**, 3, 6, T}	T_3 + Dom7 {**1**, 3, **7**, T}	T_8 + m7 {**E**, 3, 6, **8**}	T_8 + M7 {0, 3, **7**, **8**}	L_1* {**1**, 3, 6, T}
M7 {0, 4, 7, E}	T_4 + m7 {**2**, 4, 7, **E**}	T_9 + m7 {0, 4, 7, **9**}	T_1 + ø7 {**1**, 4, 7, **T**}	T_0 + Dom7 {0, 4, 7, **T**}	T_4 + m7 {**2**, 4, 7, E}	T_9 + m7 {0, 4, 7, **9**}	T_9 + Dom7 {**1**, 4, 7, **9**}	T_4 + ø7 {**1**, 4, 7, **T**}

1. T_n + [chord type] indicates a transposition of the root of the chord by n_{mod12} along with a change in the chord type. For instance, only a single pitch class changes with "T_1 + dim," e.g. C+ becomes C#°.
2. Example chords are listed in the left column, and what those samples would become is indicated in the subsequent columns of the same row. Although these are unordered pcsets, they are presented in a manner that suggests the most direct voice leading. Changed pitches are bolded, and when only one pitch is changed, the displacement is in an evenness-preserving parsimonious transformation.
3. Shading is used to group pairs of δ^* in polar opposite directions.
4. "N/A" must be expressed as $d(x, y)$, not d^*.

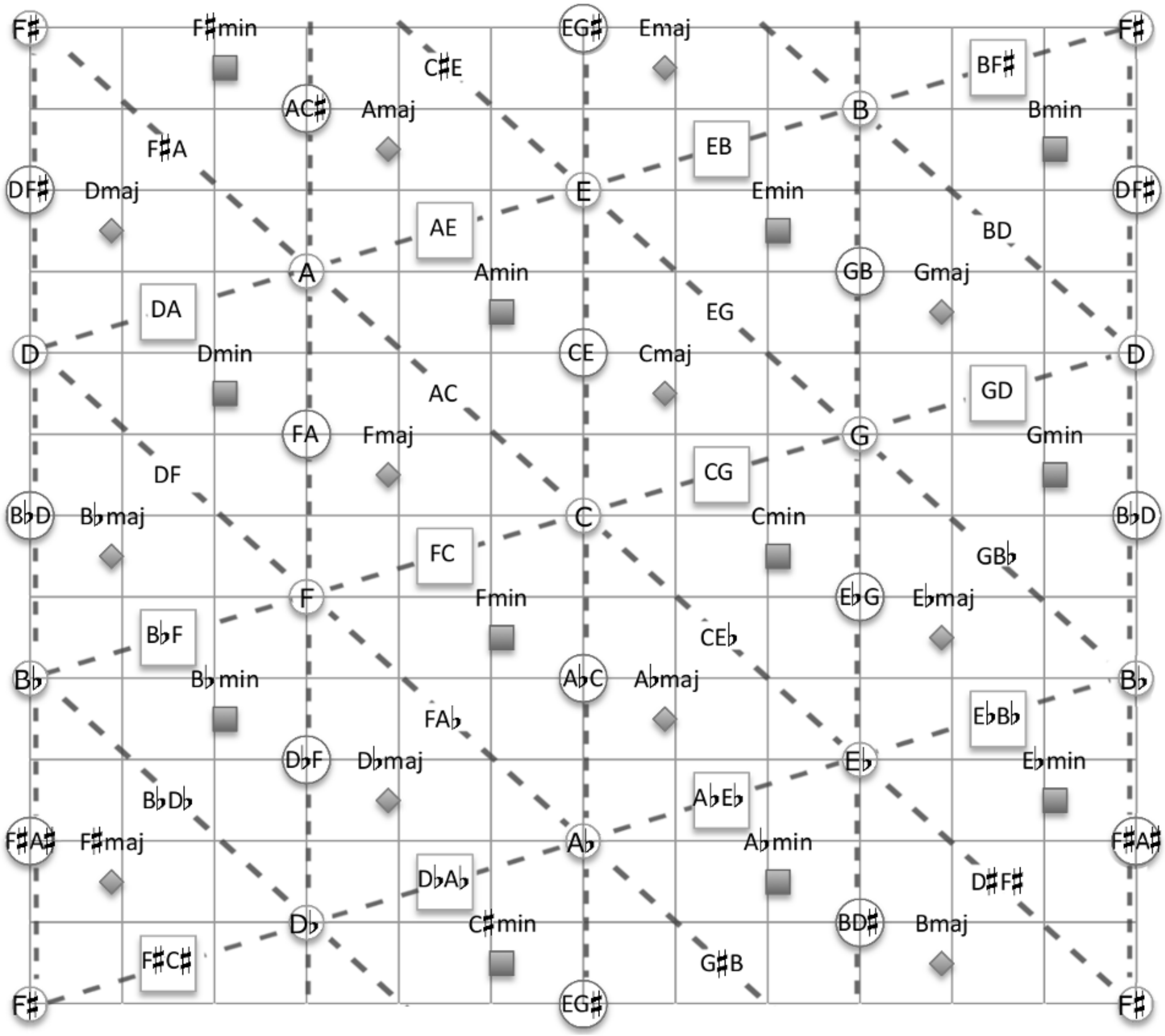
Example 38. Evenness-preserving neo-Riemannian parsimonious transformations considered in an octatonic scalar context



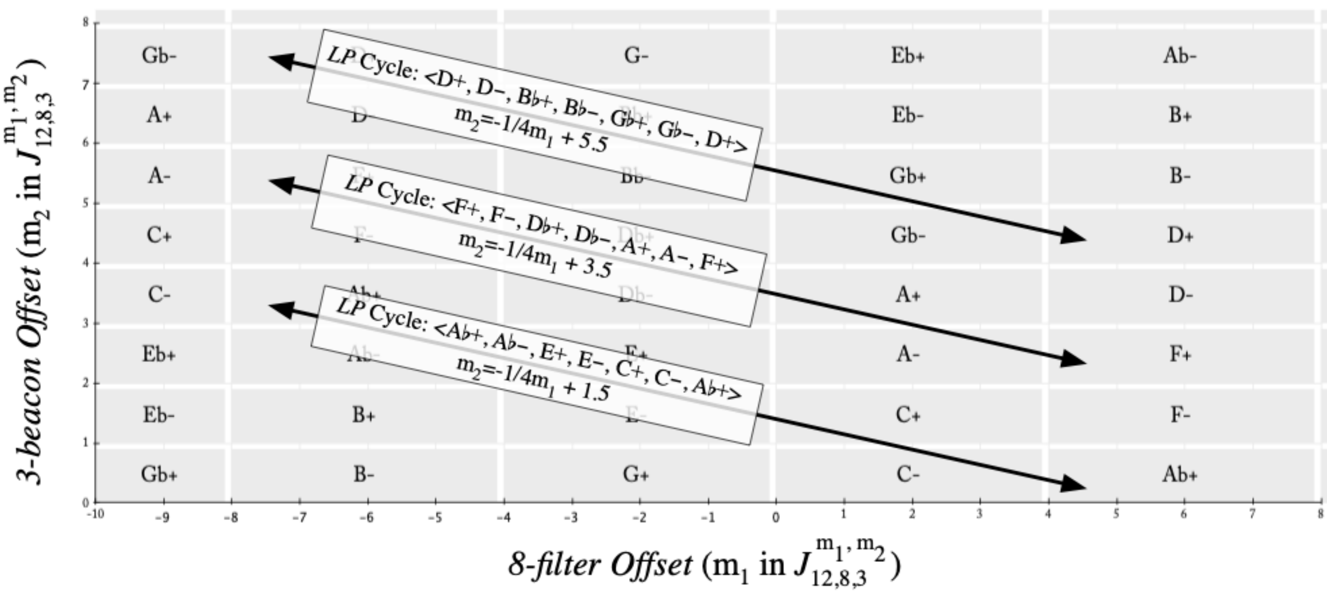
Example 39. Evenness-preserving parsimonious transformations considered in a diatonic scalar context. *P*, *L*, and *R* are colored to match the coloring in Figure 38



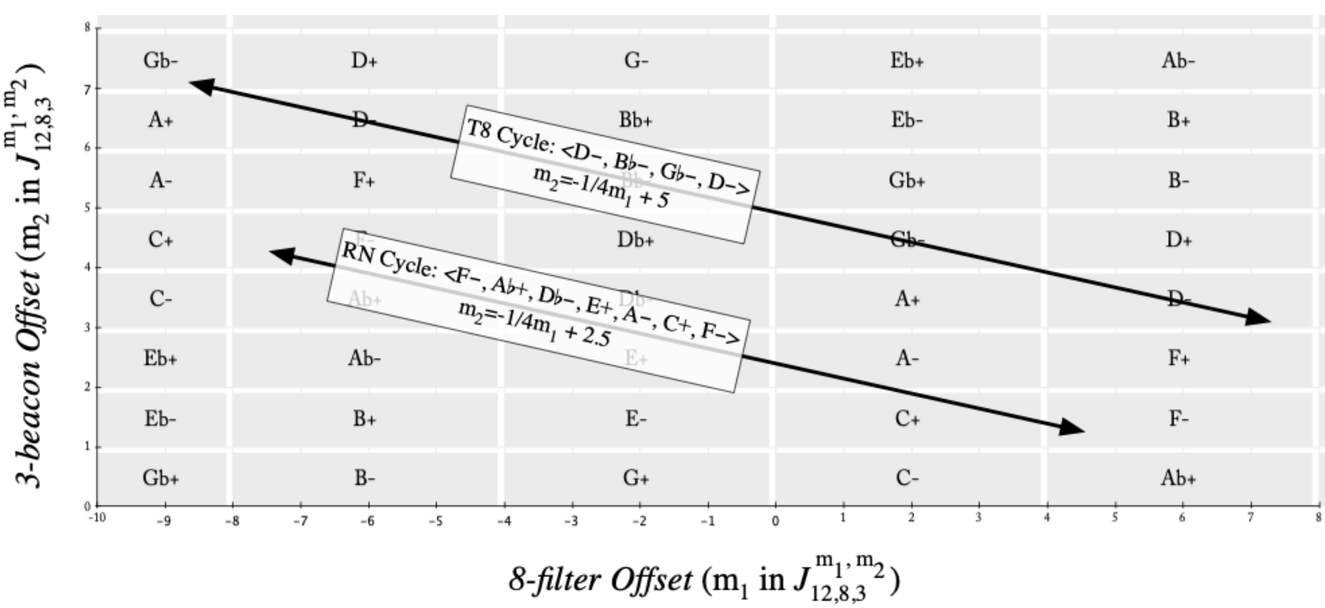
Example 40. $\varphi_{3/5}$-space from Yust 2015



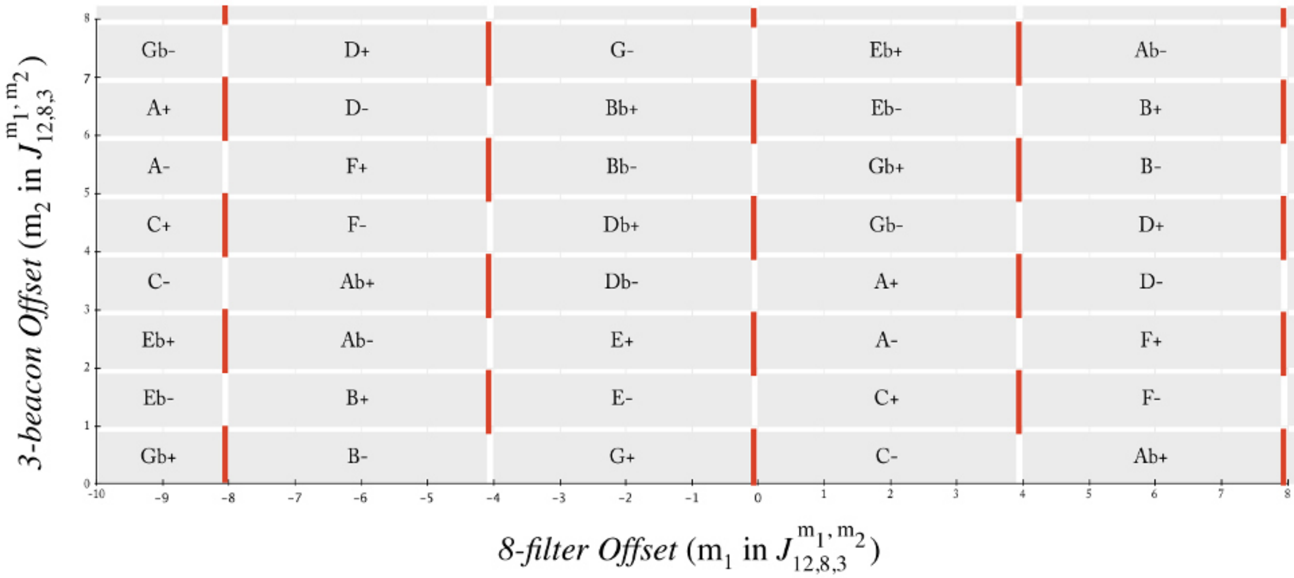
Example 41. Parsimonious cycles with negative slope in a 3→8→12 configuration space



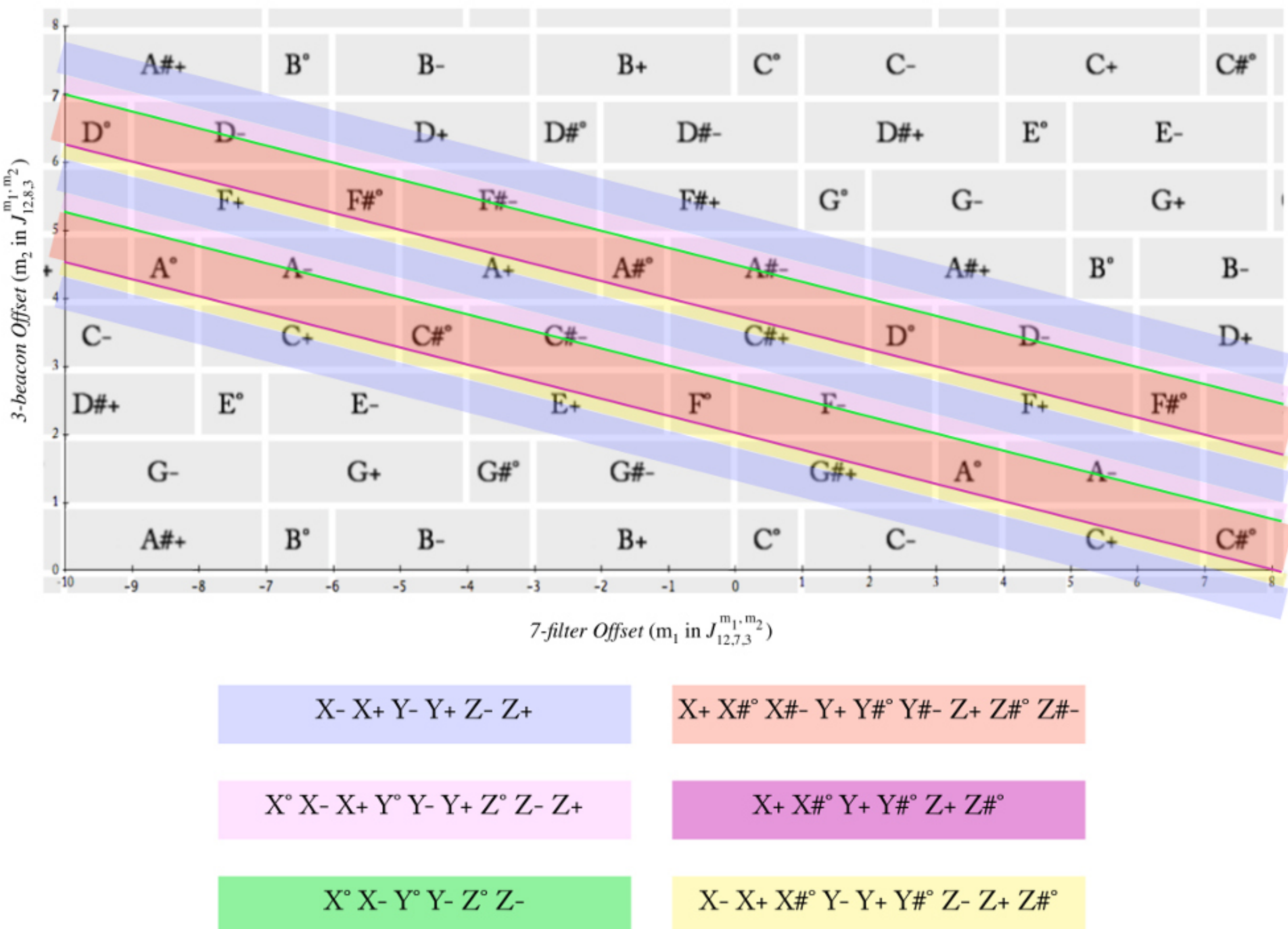
Example 42. Un-parsimonious cycles with negative slope in a 3→8→12 configuration space



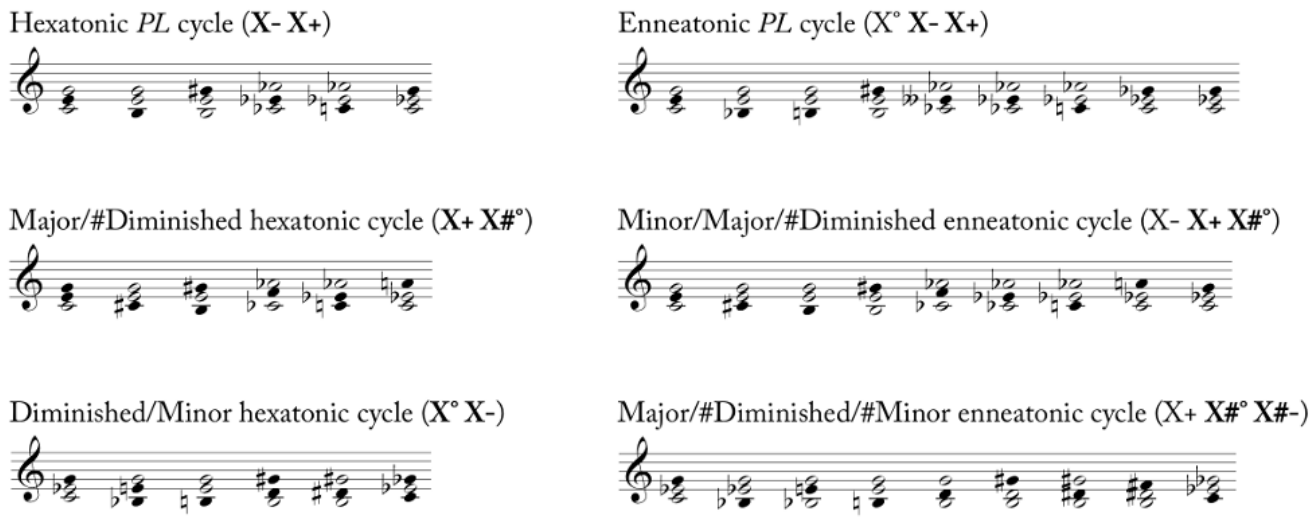
Example 43. Indicators of un-parsimonious boundaries in a 3→8→12 configuration space



Example 44. Cycles with negative slope in a 3→7→12 configuration space
Y is T4(X), Z is T4(Y), X is T4(Z)
X# is T1(X), Y# is T1(Y), Z# is T1(Z)
Y is T3(X#), Z is T3(Y#), X is T3(Z#)
Note that in all drawings of configuration spaces, including this one where alignment is crucial, it is the bottom-left edge of each region that is aligned with integer offsets. This means that each region has a bottom-left corner in gray, and is bordered along its top and right in white.



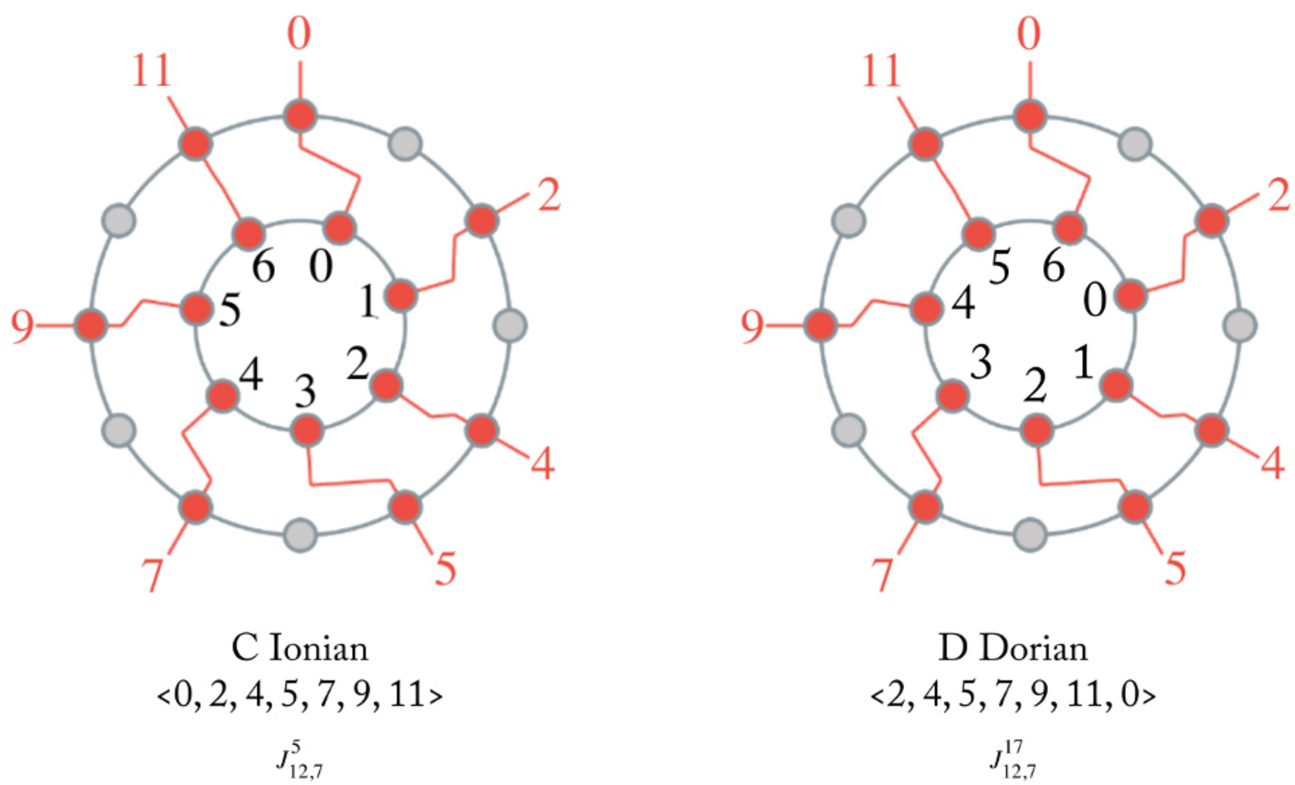
Example 45. Plot of diatonic cycles defined by a slope of 𝕄 = −¼ in a 3→7→12 configuration space



Example 46. Ordered output of C Ionian and D Dorian, at $\mathcal{J}^{5}_{12,7}$

and $\mathcal{J}^{17}_{12,7}$

, respectively.



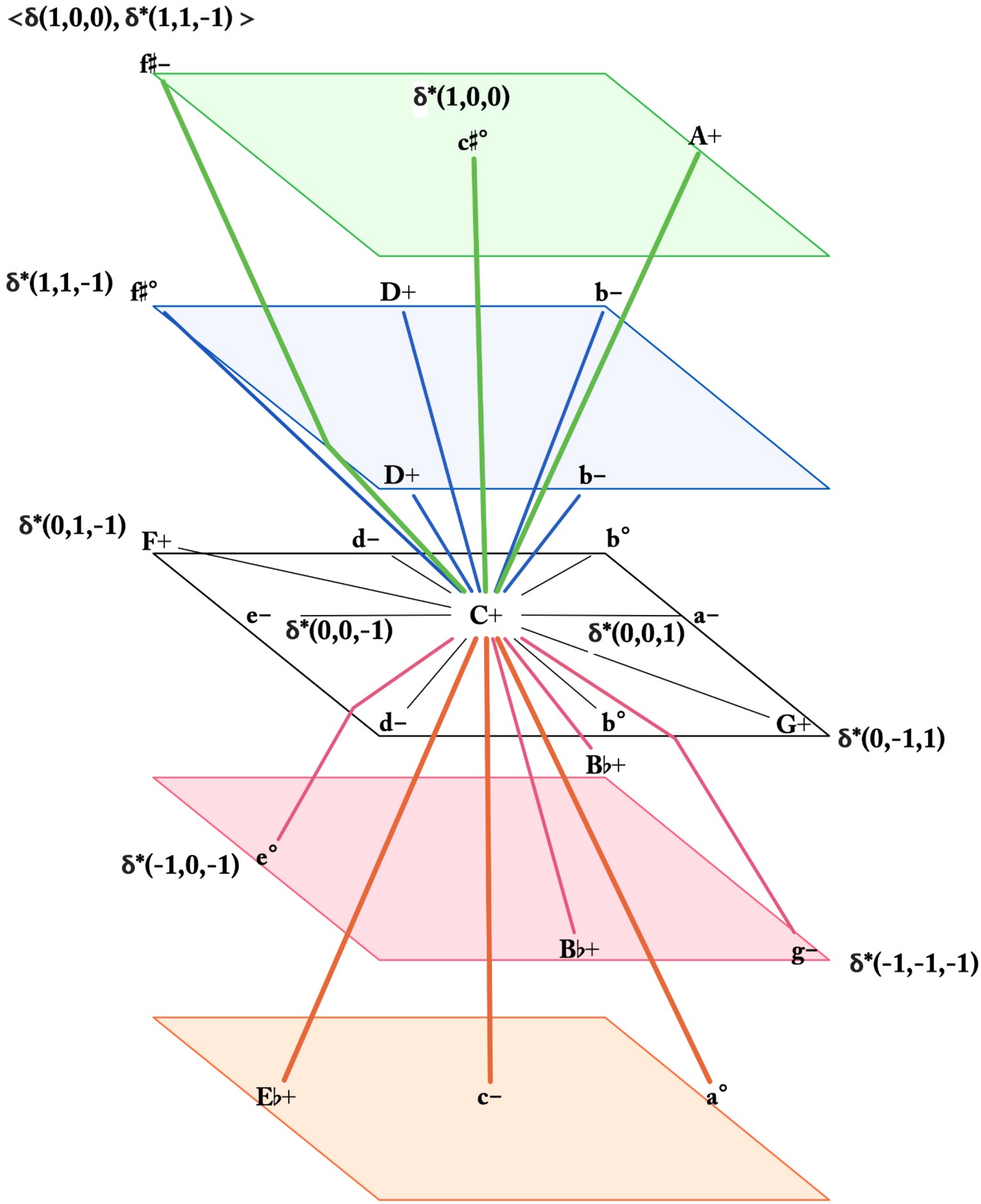
Example 47. The configuration space of the 3-beacon and inner 7-filter in a 3→7(inner)→7(outer)→12 FiPS configuration, expressed in the key of C-major with $m_1 = 5$

3-filter Offset (m_3 in $J_{12,7,7,3}^{5,\,m_2,\,m_3}$)

(C, +)	(D, -)	(E, -)	(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)
(E, -)	(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)	(E, -)	(F, +)
(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)	(E, -)	(F, +)	(G, +)	(A, -)
(B, °)	**(C, +)**	(D, -)	(E, -)	(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**
(D, -)	(E, -)	(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)	(E, -)
(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)	(E, -)	(F, +)	(G, +)
(A, -)	(B, °)	**(C, +)**	(D, -)	(E, -)	(F, +)	(G, +)	(A, -)	(B, °)
(C, +)	(D, -)	(E, -)	(F, +)	(G, +)	(A, -)	(B, °)	**(C, +)**	(D, -)

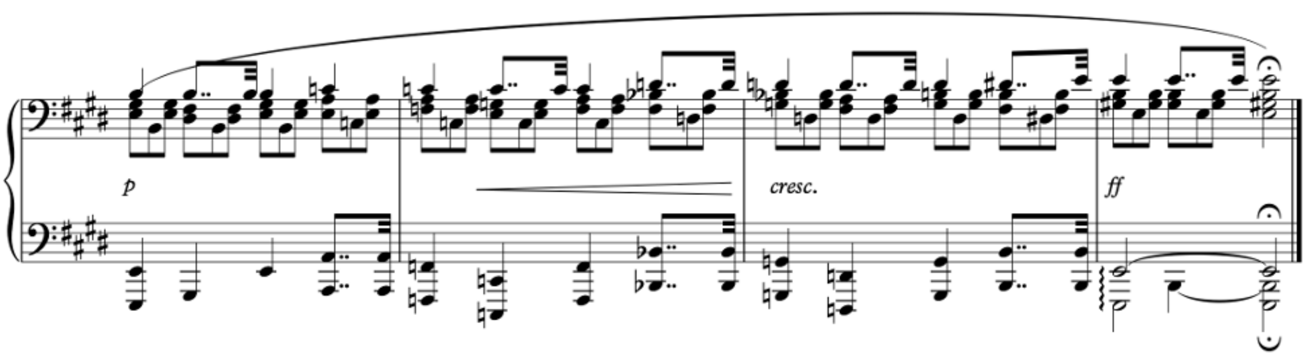
Inner 7-filter Offset (m_2 in $J_{12,7,7,3}^{5,\,m_2,\,m_3}$), moving in increments of +7

Example 48. A three-dimensional plot of 3→7(inner)→7(outer)→12 configuration space, where each vertical layer represents key changes caused by the outer 7-filter. Relationships relative to a C+ tonic chord are shown. Some labels for δ and δ*midast; have been added, to offer some perspective on the relationships between harmonies; the f#- triad at the top left corner is distinct from the f#° chord the layer below, because going from C+→f#- first involves changing from C+ as C:I to C+ as G:IV.



(Video) Example 49. A 3→7→7→12 FiPS configuration in which the inner 7-filter moves each C-major-constrained harmony up by t1

(Video) Example 50. A 3→7→7→12 FiPS configuration in which the inner 12-filter moves each harmony up by T_1

Example 51. The final phrase of Chopin's Op. 28, No. 9 Prelude in E major, mm. 9–12

Example 52. *J* representations of every chord in the final phrase of Chopin's prelude, paired with the δ* representations of the operations that incrementally change the output of FiPS. In three locations, curly braces are used to indicate the inclusion of a δ operation that changes the prevailing key without changing the pcs—we might see this as FiPS dealing with a pivot chord.

$$J^{0,9,0,6}_{12,12,7,7,3} = \{4, 8, 11\}$$

$$\xrightarrow{\delta^*(0,0,-1,1)} J^{0,9,-7,7}_{12,12,7,7,3} = \{3, 6, 11\}$$

$$\xrightarrow{\delta^*(0,0,1,-1)} J^{0,9,0,6}_{12,12,7,7,3} = \{4, 8, 11\}$$

$$\xrightarrow{\delta^*(0,-1,1,-1)} J^{0,6,7,5}_{12,12,7,7,3} = \{4, 9, 0\}$$

$$\left\{\begin{array}{l} \xrightarrow{\delta^*(0,-1,0,1)} J^{0,5,7,6}_{12,12,7,7,3} = \{5, 9, 0\} \\ \xrightarrow{\delta(0,-1,0,0)} J^{0,4,7,6}_{12,12,7,7,3} = \{5, 9, 0\} \end{array}\right.$$

$$\xrightarrow{\delta^*(0,0,-1,1)} J^{0,4,0,7}_{12,12,7,7,3} = \{4, 7, 0\}$$

$$\xrightarrow{\delta^*(0,0,1,-1)} J^{0,4,7,6}_{12,12,7,7,3} = \{5, 9, 0\}$$

$$\xrightarrow{\delta^*(0,0,1,-1)} J^{0,4,14,5}_{12,12,7,7,3} = \{5, T, 2\}$$

$$\left\{\begin{array}{l} \xrightarrow{\delta^*(0,0,0,1)} J^{0,4,14,6}_{12,12,7,7,3} = \{7, T, 2\} \\ \xrightarrow{\delta(0,-1,0,0)^2} J^{0,2,14,6}_{12,12,7,7,3} = \{7, T, 2\} \end{array}\right.$$

$$\xrightarrow{\delta^*(1,-1,-1,1)} J^{12,1,7,7}_{12,12,7,7,3} = \{6, 9, 2\}$$

$$\left\{\begin{array}{l} \xrightarrow{\delta^*(0,-1,1,-1)} J^{12,0,14,6}_{12,12,7,7,3} = \{7, E, 2\} \\ \xrightarrow{\delta(0,-1,0,0)^2} J^{12,82,14,6}_{12,12,7,7,3} = \{7, E, 2\} \end{array}\right.$$

$$\xrightarrow{\delta^*(-1,-1,1,-1)} J^{0,81,21,5}_{12,12,7,7,3} = \{6, E, 3\}$$

$$\xrightarrow{\delta^*(0,0,1,-1)} J^{0,81,21,5}_{12,12,7,7,3} = \{8, E, 4\}$$

(Video) Example 53. The final phrase of Chopin's prelude, represented by a 3→7→7→12 FiPS configuration

(Video) Example 54. The material from Example 53, isolating the rotations of the 3-beacon

(Video) Example 55. The material from Example 53, isolating the rotations of the 3-beacon and inner 7-filter

Example 56. A plot of the rotational offsets of the 3-beacon and inner 7-filter from Example 53