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Pentatonic *Xuangong* 旋宮 Transformations in Chinese Music

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ABSTRACT: The anhemitonic pentatonic scale is fundamental to Chinese music theory, and so is the concept of *xuangong*: transformations from one pentatonic scale to another. The vocabulary related to these transformations is as diverse as the musical contexts in which it appears; similar moves can be described using a multitude of perspectives, resulting in overlapping and, at times, confusing terminology. To describe *xuangong* transformations, I adopt the precise language of signature transformations to enrich, complement, and shed light on Chinese music theory. The four basic *xuangong* transformations are chromatic transposition (C, D, E, G, A → G, A, B, D, E), pentatonic transposition (C, D, E, G, A → G, A, C, D, E), *bian*-directed transformation (C, D, E, G, A → B, D, E, G, A), and *qing*-directed transformation (C, D, E, G, A → C, D, F, G, A). The last two are adapted from classical Chinese music theory, and they are analogous to key signature transformations in Western music. This paper discusses the structure of *xuangong* transformations, their application in Chinese music theory, and their analytical use in examples spanning Confucian court music, traditional instrumental music, Cantopop, and Chinese new music, both tonal and atonal.

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1. Introduction

[1.1] The anhemitonic pentatonic scale is fundamental to Chinese music theory, and so are the transformations from one pentatonic scale to another (Du 2019; Du and Qin 2007, 200–233). These transformations are known variously as *xuangong* 旋宮, modulation (*zhuandiao/fandiao* 轉調/犯調), or the theory of 60 modes (*liushidiao* 六十調).(1) The terms collectively describe modal relations involving the cycles of five pentatonic positions, of twelve chromatic pitch classes, and of their product (12 × 5 = 60). Yet, despite this structural simplicity, *xuangong* transformation is one of the knottiest areas of Chinese music theory. As Liu Yongfu 劉永福 describes,

對於 “旋宮轉調” 的理論表述, 歷來含混不清、令人費解。形形色色的概念 、術語, 如同層層迷霧, 遮擋著人們的 “視線”, 束縛著人們的思維, 阻礙著學科的建設, 致使 “普通人” 至今對此難以理解和認知。 (Liu 2005, 102)

The theoretical expression of *xuangong zhuandiao* has always been ambiguous and puzzling. Like layers of fog, the different kinds of concepts and terms obscure “sight,” shackle thought, and hinder the field’s advancement; as a result, it is still difficult for “laypeople” to understand and comprehend.

[1.2] Recent developments in mathematical music theory—particularly signature transformations (Hook 2008, 2023; Tymoczko 2005, 2020, 2023; Lam 2020)—have lent new theoretical and analytical perspectives to scalar collections. In the aesthetic framework of European classical music, the pentatonic scale has long been conceptualized as primitive and exotic (Lee 2020); mainstream textbooks today continue to perpetuate the view that the pentatonic scale is harmonically static and deficient in some way (Yu Wang 2020, para. 8). In this essay, I examine Chinese pentatonicism on its own terms, celebrating the richness and depth of Chinese music in its myriad forms. I explore how Western scale theory can shed light on foundational Chinese theory, how recent ideas in Western scale theory are implicit in centuries-old musical thought, and how a cross-cultural music theory operates in concept and practice.

[1.3] Set to tango rhythms popular at the time, Ren Guang’s 任光 well-known tune “Choiwan jeui yyut” 彩雲追月 (1935) demonstrates phrase-level melodic transformations.(2) **Example 1.1** (**Audio Example 1**) shows a Cantopop version of the tune that stays entirely within D pent. (DEF♯AB). The tune opens with a stepwise pentatonic ascent and continues into a sentential parallel period. The consequent transposes the antecedent down by two pentatonic steps (the A–B dyad near the end moves down an octave). Notably, under this stepwise transposition, the recurring eighth-note dyad shrinks from a m3 (A–F♯) in the antecedent to a M2 (E–D) in the consequent. There is one extra transformation in Ren Guang’s original instrumental version (**Audio Example 2**) that the vocal version leaves out: the D → C♯ semitone alteration in m. 9. This semitone displacement is the result of a transformation applied to m. 9 as a whole, not just the individual note: i.e., a minimal voice leading from one pentatonic scale to another (D, E, F♯, A, B to A, B, C♯, E, F). **Example 1.2** isolates the syncopated motif, and it shows that the combined effect of the stepwise shift and semitone shift is the same as T_7.

[1.4] The three transformations in Example 1.2 belong to the group of *xuangong transformations*, and they are the pentatonic versions of Julian Hook’s (2008, 2023) diatonic *signature transformations*. For comparison, **Example 1.3** shows a similar network of diatonic transformations in a waltz by Franz Schubert—moving up three steps, adding one sharp, and T_7. In staff notation, diatonic transformations are more intuitive than pentatonic ones, since note names and key signatures naturally highlight diatonic relations. Tymoczko’s (2005, 2020, 2023) subsequent generalization of signature transformations to other scales promises wider application beyond the diatonic scale. Yet the way signature transformations manifest in music and music theory differs widely for each scale type. *Xuangong* theory is one such case. The following two sections will define *xuangong* transformations and how they intersect with Chinese music theory.

Part I. Theory

2. Chromatic and pentatonic transpositions

[2.1] Pentatonic *xuangong* transformations act on two elements recognized in ancient Chinese music theory (**Example 2.1**): twelve *lülü* 律呂, which are similar to pitch classes, and five pentatonic (scale-step) positions: **gong** 宮, **shang** 商, **jue** 角, **zhi** 徵, **yu** 羽 (Example 2.2a). By convention, position **gong**, the root or origin of fifth generation (**Example 2.2b**), represents the entire pentatonic set (*yun* 均) regardless of the mode (*diao* 調) or final (*tou* 頭). For example, the set DEF♯AB is always “D pent.,” no matter what the modal final is.(3)

[2.2] Formally, *xuangong* transformations act on a *pentatonic pitch class*, which is the ordered pair

(pc_{pent}, pos_{note}).

The first element, pc_{pent}, is a mod-12 pc representing the pentatonic set by its **gong**.[(4)] For example, all notes in Example 2.2 belong to D pent., so all $pc_{pent} = 2$ (assigning **gong** to D). The second element, pos_{note}, is a pentatonic position.[(5)] Index numbers 0 to 4 (mod 5) represent positions **gong** to **yu** in step order. The two elements together describe the pentatonic context for each note, pc_{note}, which is not directly represented in the ordered pair. For example, (D, **yu**), or (2, 4), describes position **yu** in D pent., so the actual note is B. There are 60 different pentatonic pcs and as many *xuangong* transformations, discussed in detail below.

[2.3] *Xuangong* means "to rotate **gong**," or for a lülü "to rotate as **gong**." *Liji* 禮記 (*Book of rites*), supposedly compiled by Confucius (551–479 BCE), contains the earliest mention of *xuangong* (in its long form *xuanxiang wei* **gong** 還相為宮). In the Confucian cosmology of *Liji*, musical pitches exemplify nature's cyclic and harmonious order:

Original	Concise translation (mine)	Expanded translation (Legge 1885, 382)
五行、四時、十二月，還相為本也。	Five elements, four seasons, twelve months, rotate as the origin.	The five elements in their movements alternately displace and exhaust one another. Each one of them, in the revolving course of the twelve months of the four seasons, comes to be in its turn the fundamental one for the time.
五聲、六律、十二管，還相為宮也。 (Zheng 1980 [East Han], 1423) [(6)]	Five notes, six *lü*, twelve pipes, rotate as position **gong**.	The five notes of harmony, with their six upper musical accords, and the twelve pitch-tubes, come each, in their revolutions among themselves, to be the first note of the scale.

[2.4] The cyclical idea was later realized as *xuangong* tone wheels in Yueshu yaolu 樂書要錄 (Yuan 2021 [ca. 680], 86) to describe modes in the abstract. My tone wheels in **Example 2.3** have been substantially simplified. The two rings of the tone wheel correspond to the two elements of a pentatonic pitch class defined in [2.2]. The outer ring is the pitch-class clock face, the inner ring shows the five positions, and the gray labels show notes in one of the diatonic supersets (discussed in the next section). The arrows are my own addition. The blue arrow attached to **gong** affixes pentatonic positions in the inner ring to specific pcs/*lülü* in the outer ring. The red arrow in the middle locates the modal final within that pentatonic set. By default, tone wheels are set to *huangzhong* (C) and **gong** at 6 o'clock; see **Example 2.3a**. **Example 2.3b** shows position **yu** in D pent., (2, 4). The blue arrow points to *taicu* (D) and the red arrow points to **yu**, and the corresponding pc/*lülü* of **yu** is the modal final *yingzhong* (B). Below, I will examine transpositions that act on pc_{pent} and pos_{note} (each ring) individually and discuss examples in Chinese music where they perform structural roles. Then, I will discuss how the two transpositions interact.

[2.5] *Xuangong* transformations are structural to many genres. In *guchuiyue* 鼓吹樂—percussion-and-shawm music that accompanies funeral and temple rites (Li 1985; Jones 1995, 2007, 2010)—the chromatic transposition T_n takes on extraordinary significance.[(7)] T_n transposes pc_{pent} by n semitones mod 12. The pentatonic position remains unchanged:

$$T_n(pc_{pent}, pos_{note}) = ((pc_{pent} + n), pos_{note}),$$

$$T_{12} = T_0.$$

Example 2.4 shows an excerpt of the *qupai* 曲牌 (fixed tune) called "Shuilong yin" 水龍吟 ("Water dragon chant") played by the Hua 滑 family band from Yanggao, Shanxi province (Jones 2007, 91–99). The Hua family band plays the tune first in the home key, and then T_{10} transposes the home key into the plum blossom key.[(8)] (Poetic nomenclature like this is not uncommon.) A few things obscure the straightforward T_{10}: improvised ornaments, non-pentatonic notes, and most

importantly, octave shifts. The bell tone (lowest and loudest note) of the shawm is E, so at T_{10}, players shift the resultant low D (and only that note) up an octave, creating large leaps and new melodic contours. Recognizing the *qupai* relies on one's ability to identify invariant pentatonic positions under T_n.

[2.6] The pentatonic transposition t_n transposes pos_{note} by n steps modulo 5 without affecting pc_{pent}:

$$t_n(pc_{pent}, pos_{note}) = (pc_{pent}, (pos_{note} + n)),$$

$$t_5 = t_0.$$

In other words, t_n describes transpositions inside the same pentatonic set.

[2.7] The pentatonic upper fifth, t_3, is a structural transposition in several regional styles, including Cantonese opera, *beiguan* 北管, and Chaozhou zheng music. In Chaozhou zheng music, the *qupai* titled "Guo jiang long" 過江龍 ("River-crossing dragon") alternates between *zhengxian* 正線 (home key) and *fanxian* 反線 (inverse key), which is t_3 of the home key. The first note in **Example 2.5** transposes like so:

$$t_3(5, 3) = (5, (3 + 3)) = (5, 1).$$

[2.8] In heptatonic notation, whether it is *gongche* notation 工尺譜, *jianpu* 簡譜 (localized Western solfège), or Western notation (see Example 2.1 above), pentatonic transformations are more challenging to identify than diatonic ones. Complexity arises from the fact that a given pentatonic interval can be one of two diatonic intervals; for example, one pentatonic step is 2 or 3 semitones, which could be a diatonic M2 or m3. In Example 2.5, all notes in the t_3 of *Guo jiang long* move by a P5 except for A → F, which moves by a m6. The table in **Example 2.6** lists all corresponding pentatonic and diatonic intervals in F pent.

[2.9] In Chinese new music, t_n can create rich contrapuntal relations. Duan Pingtai 段平泰 (2013, 171) teaches canons that imitate at pentatonic t_n while adhering to diatonic harmony. In **Example 2.7**, Duan's neo-Renaissance canon imitates at three pentatonic steps below ($t_{-3} = t_2$), and it stays strictly within F pent. This canon is diatonically consonant every half note. But if the imitation were at T_{-7} instead of t_{-3}, then the A in m. 4 would be B♭, forming a dissonant interval with the C above. In conventional canons, diatonic transposition does not alter consonance and dissonance (except in the case of P5/d5). However, in Duan's pentatonic canon, it is harder to predict whether a pentatonic step is dissonant (M2) or consonant (m3), making this kind of counterpoint more complex and labor intensive than its purely diatonic counterparts.

3. Bian- and qing-directed transformations

[3.1] The pentatonic scale shares properties with its complement, the diatonic scale (Clough et al. 1999, 101–2). The *bian*- and *qing*-directed transformations are analogs of flatwise and sharpwise signature transformations, where each accidental in a key signature represents a semitonal displacement between scales a fifth apart. A review of pentatonic "accidentals" will precede formal definitions.

[3.2] *Zhengyin* 正音 refers to the proper pentatonic positions, and *pianyin* 偏音 refers to their semitone alterations. The *pianyin* prefix 變 *bian* (lit. change) lowers the position by one semitone; the opposite, 清*qing* (lit., clarity), raises the position by one semitone. Therefore, the C major diatonic set is **gong**, **shang**, **jue**, **qingjue**, **zhi**, **yu**, **biangong** (**Example 3.1a**), or, in letter names, C, D, E, E-*qing*, G, A, C-*bian*.[9] **Biangong** and **qingjue** are privileged *pianyin*, because they are adjacent to the *zhengyin* of C pent. in fifth order (**Example 3.1b**).

[3.3] Readers may encounter at least six definitions of *bian* in relation to *xuangong*.

1. *Bian*, in the most general sense: change or alteration (seen in Examples 3.3, 6.3, and 6.4).
2. *Bianhuan lun* 變換論 as the Chinese translation of Lewinian transformational theory.
3. *Bianyin* 變音 as an alternate to *pianyin* 偏音, encompassing semitone alterations of pentatonic positions in both directions.
4. *Bian* as a prefix to a pentatonic position (e.g., *biangong*), indicating its lowering by semitone.
5. b_n as a generalized *bian*-directed transposition described below.
6. *Hen* 変 as a Japanese loanword from Chinese theory, the flat accidental in Western music.

Therefore, depending on the context, *bian* can mean alterations in general (definitions 1–3), or specifically a lowered semitone (definition 4). While the Chinese term for flat is simply "to lower" (*jiang* 降), in Japanese, the loanword *hen* (definition 6) also acts upon diatonic letter names.

[3.4] **Example 3.2** illustrates *bian*- and *qing*-directed transformations adapted from Chinese theory. They operate on both elements of a pentatonic pitch class as defined in [2.2] above, that is, on (pc_{pent}, pos_{note}). The transformation labeled b_1 refers to one specific *bian*: it lowers **gong** ($pos_{note} = 0$) by a semitone to **biangong** and then reinterprets it as **jue** ($pos_{note} = 2$) of the new scale. In Chinese, this move is shortened to "**biangong** as **jue**" 變宮為角.(10) Similarly, q_1 is "**qingjue** as **gong**" 清角為宮, which raises **qing** ($pos_{note} = 2$) to **qingjue** and then reinterprets it as **gong** ($pos_{note} = 0$) of the new scale. The two transformations are inverses of each other:

biangong → **jue**
gong ← **qingjue**

Readers may appreciate a helpful memory aid here: "*b*" resembles a flat, and "*q*" is "*b*" upside-down.

As Example 3.2 shows, "**biangong** as **jue**" and "**qingjue** as **gong**" transpose both pc_{pent} and pos_{note} in a specific way. The note (C, gong), or (0, 0) is transformed like so:

$$b_1(0, 0) = (7, 2),$$
$$q_1(0, 0) = (5, 3).$$

[3.5] Generalizing this, the *bian*-directed transformation b_n is the repeated application of "*biangong* as jue" n times.(11) The first element ($pc_{pent} + 7n$) is calculated mod 12, and the second element ($pos_{note} + 2n$) is calculated mod 5:

$$b_n(pc_{pent}, pos_{note}) = ((pc_{pent} + 7n) \bmod 12, (pos_{note} + 2n) \bmod 5).$$

Likewise, the *qing*-directed transformation q_n is the repeated application of "**qingjue** as **gong**" n times:

$$q_n(pc_{pent}, pos_{note}) = ((pc_{pent} + 5n) \bmod 12, (pos_{note} + 3n) \bmod 5),$$

The inverse of b_n is q_n:

$$b_1 = (q_1)^{-1} = q_{-1},$$

$$q_1 = (b_1)^{-1} = b_{-1}.$$

Sections below will explore a few important concepts in Chinese theory underpinned by q_1.

Qupai variations

[3.6] "Baban" 八板 ("Eight beats"), an iconic *qupai*, has a bewildering assortment of titles and variations (Thrasher 1989, 73–82, 2016; Qian 1990, 9), and a notable subset of these variations are related by $q_{\pm 1}$.(12) **Example 3.3** and **Audio Example 5** compare two variants of "Baban": "Lao liuban" 老六板 ("Old six beats") and "Jinshe kuangwu" 金蛇狂舞 ("Wild dance of the golden snake"). The B section of "Jinshe" is largely q_1 of "Lao liuban". This variation technique is called *gefan* 隔凡 or *bianfan* 變凡 (skip or change *fan*), as q_1 raises the *gongche* solfège from *gong* 工 to *fan* 凡 (the solfège equivalent of mi → fa).(13) Many pipa players are probably acquainted with this particular q_1, since both versions are standard exam pieces (Central Conservatory of Music 2013). A similar q_1 variation with rhythmic augmentation is known as "Fanwang gong" 梵王宮 ("Brahma's palace" (**Audio Example 6**). The Buddhist title, meaning "Brahmā's palace," is a pun on "*fan* replaces *gong*" (*fan wang gong*凡忘工), or, in diatonic solfège, "*fa* replaces *mi*."

Shawm fingering and qin tuning

[3.7] In terminology for shawm fingering and qin tuning, b_n and q_n serve as pentatonic "key signatures" in the sense that they describe pentatonic sets as semitone alterations of an original set without specifying a tonic. In shawm termniology, *jiezi* 借字 (borrowed notes) describes the number and direction of semitone alterations from the shawm's home key. The table in **Example 3.4** shows terms adopted by musicians in Liaoning province (after Yang 1996; Lin 2011). Half-hole and fork fingerings limit the number of scales to seven, with three scales on either side.(14) On the *qing*-side, the term jie 借 counts the number of borrowed notes; on the *bian*-side, the term *yashang* 壓上 describes the successive lowering of the *gongche* position *shang* (the solfège equivalent of **do**).(15) For example, b_2 of the home key (C pent.) is called "lower *shang* twice," because the two alterations in D pent. are G → F♯ and C → B.

[3.8] The standard tuning (*zhengdiao* 正調) of the qin's seven strings are arranged in pentatonic steps (**Example 3.5**). A core subset of alternate tunings (*waidiao* 外調) are $b_{\pm n}$ of the standard tuning (Thompson n.d.a). In this context, *man* 慢 (loosen) and *jin* 緊 (tighten) are equivalents of *bian* and *qing*, respectively. While the naming schemes are not entirely consistent, each has its own logic. For example, *manjuediao* is b_1 of the standard tuning. The name indicates that the new **jue** is a result of the loosened (*man*) **gong**, which is another way of saying "**biangong** as **jue**." Each tuning is associated with tuning instructions that closely resemble a key signature, such as "loosen the third [string]" for *manjuediao*.

Piano etudes

[3.9] In Chinese new music, *bian*- and *qing*-directed transformations continue to be a vital compositional resource. In his children's suite *Yigong Variations* (2000), for example, Li Yinghai 黎英海 explores all five modes on D via four successive q_1 transformations (**Example 3.6**).(16) This suite stems from earlier piano etudes that cycle through all 60 modes. He wrote them in 1964 in response to new pentatonic music, so that pianists can "gain familiarity with the scale steps of each scale, mode, and common melodic figures" (2002, i).(17) **Example 3.7** excerpts an opening exercise Li titled "parallel-relation, **gong**-transposing, mode-changing exercise (gradual transition), upshift" (2002, 3), or what one could call a q_1 sequence. Each system groups five parallel modes together, and pianists can link up the q_1 sequence by playing one system after another.

[3.10] In the exercise, the pianist arrives at T_1 of the opening scale after five iterations of q_1, therefore

$$q_5 = T_1 \; ; \; b_5 = T_{11}.$$

Similarly, after twelve iterations of q_1, the pianist returns to C pent., one step higher (t_1) than the original, consequently,

$$q_{12} = t_1; b_{12} = t_{-1} = t_4.$$

Li's full exercise eventually iterates through all 60 modes, and it shows that b_1 or q_1 singularly generates all 60 transformations (including T_n and t_n). The index numbers of q_n and b_n are calculated mod 60:

$$q_{60} = b_{60} = q_0 = b_0.$$

[3.11] While *bian* and *qing* are similar to changes in diatonic key signatures, I have avoided using Hook's (2008) sharpwise (s_n) and flatwise (f_n) notation, since diatonic accidentals conflate the direction of fifth transposition and semitone inflection. For diatonic transformations, the key signature sharp is associated with congruent upward transposition: the underlying scale transposes up by P5 via an upward semitone inflection (**Example 3.8**). However, the pentatonic b_n moves the underlying scale up a fifth by a downward semitone inflection, effectively disassociating directionality from accidentals. The contrast stems from the complementary relation of diatonic and pentatonic sets. Consider the complementary sets C dia. and F♯ pent., the white and black keys on a piano, on the circle of fifths (**Example 3.9**). Transposing both sets by a fifth—one click clockwise—swaps two notes. The diatonic set swaps F for F♯, an upward alteration; the pentatonic set reciprocally swaps F♯ for F/E♯, which is a downward alteration. In short, complementary sets have inverse semitone inflections in relation to fifth transposition.

[3.12] I will conclude this section by revisiting "Choiwan jeui yyut"'s motivic network. The cumulative transformation in **Example 3.10** can be written as $t_3b_1 = T_7$. To verify this arithmetically, we can substitute each transformation with b_n. *Xuangong* transformations commute; for example, $t_3b_1 = b_1t_3$. By formula [6] above, $b_1 = T_7t_{2,\ so}$

$$b_1t_3 = T_7t_2t_3 = T_7.$$

By rearranging the equation above,

$$T_7(b_1)^{-1} = T_7q_1 = t_3.$$

Readers may trace these pathways in **Example 3.11**, which shows all fifth transpositions of a C pent. scale. **Example 3.12** expands the diagram to show all 60 forms of the C pent. scale.

4. Historical terminology

[4.1] In the sections above, I have applied *xuangong* transformation to a wide range of musical structures, from melodic segments to entire pieces, and have extended conventional *xuangong* theory beyond its historical application to mode. As I explore below, contemporary scholars continue to work on foundational *xuangong* modal theory, and in this regard, my theory intersects with and sheds light on sustained debates on the labeling of modes, rotations, and modulations. To narrow the analytical object to modes only, rather than any kind of note, I will replace the symbols pos_{note} and pc_{note} with pos_{final} and pc_{final}, which denote the modal final as the analytical object in the subscript.

Labeling modes: *weidiao/zhidiao* 為調之調.

[4.2] Historically, pentatonic modes have been represented both as (pc_{pent}, pos_{final}) or (pc_{final}, pos_{final}).[(18)] Consider these two modes:

> for D, E, G, A, C, the pc_{pent} is C, the pc_{final} is D, and the pos_{final} is **shang**,
>
> for C, D, F, G, B♭, the pc_{pent} is B♭, the pc_{final} is C, and the pos_{final} is **shang**.

In Song-era treatises, the mode label "C **shang**" could mean either of the two modes above, and the mixed use created a "confusion of modal terminology . . . probably unparalleled in the entire musical history of China" (Pian 1967, 43).[(19)] Hiyashi Kenzō 林 謙三 (1936) disentangled the naming schemes as *weidiao* 為調and *zhidiao* 之調 systems (Pian 1967, 55).

> *Wei* 為, meaning "as," disambiguates "C **shang**" as "C as **shang**" (*huangzhong wei* **shang** 黃鐘為商). This naming scheme corresponds to (pc_{final}, pos_{final}) so that "C **shang**" refers to CDFGB♭.
>
> *Zhi* 之, the possessive, disambiguates "C **shang**" as "**shang** of C pent" (*huangzhong zhi* **shang** 黃鐘之商). This naming scheme corresponds to (pc_{pent}, pos_{final}) so that "C **shang**" refers to DEGAC.

[4.3] In the last century, Chinese scholars have standardized pentatonic mode labels with the *weidiao* system (pc_{final}, pos_{final}), since these labels correspond to Western mode names like "C Dorian" (Chen 2002, 115). For instance, in "C **shang**," C is the final. The present transformations, however, act upon *zhidiao* coordinates (pc_{pent}, pos_{final}), since *weidiao* cannot directly account for pentatonic transformations (t_n). The fact that one could label modes using *weidiao* coordinates and define transformations using *zhidiao* coordinates reflects the inherent richness of the system and the need for precise language.

Labeling rotations: *zuo xuan*/*you xuan* 左旋/右旋.

[4.4] One can describe a mode by tone-wheel rotations from its default (0, 0) state. Scholars have debated the precise meaning of historical terms related to rotation, such as "left," "right," "forward," and "backward" (zuo xuan 左旋, you xuan 右旋, shun xuan 順旋, ni xuan 逆旋), respectively where ambiguity arises from different frames of references and the inherent flexibility of literary Chinese.[(20)] Much emphasis has been put on these terms as they define the very actions (*xuan*) of *xuangong*.[(21)]

[4.5] Following Huang (1993, 109–127), here is a basic interpretation. In Example 2.3, the outer ring of twelve *lülü* (or pc) is stationary, while the inner ring of pentatonic positions is movable. *Huangzhong* and **gong** are positioned at 6 o'clock by default. "Leftward" or "forward" rotation of the pentatonic ring corresponds to a *zhidiao* perspective. For the transformation C, D, E, G, A → B, D, E, F♯, A in Example 2.3, viewed head-on, **gong** rotates "leftward" (clockwise) to point to *taicu* (D), and then the *lülü* of the modal final is read from the desired pentatonic position. "Rightward" or "backward" rotation corresponds to a *weidiao* perspective. For the same transformation in Example 2.3, the modal final **yu** rotates to the *lülü* of the modal final, *yingzhong* (B). Both methods describe the same mode BDEF♯A from different frames of reference.

Labeling modulations: *xuangong*/*fandiao* 旋宮犯調.

[4.6] A debate erupted in the 2000s in an effort to clarify the meaning of two synonyms *xuangong* (rotating as **gong**) and *fandiao* (breaching the mode), both terms roughly meaning modulation (Chen 2002; Du 2003; Liu 2005; Qin 2006). In scholars' efforts to unite overlapping terminology across millennia, the results are inevitably inconclusive. My own view aligns most closely with Qin Dexiang's (2006) mathematically informed proposal:

> T_n is *xuangong*, since it changes pc_{pent}, which is literally represented by **gong**;
>
> t_n is *fandiao*, since it changes pos_{final}, which represents *diao*, the mode; b_n (or q_n) is *xuangong fandiao*, since it changes both pc_{pent} and pos_{note}.

The persisting ambiguity in labeling modes, rotations, and modulations illustrates the relational complexity at the heart of pentatonic transformations. While these issues may seem abstract, they concern foundational definitions of "xuan" and of modal relations.

Part II. Analysis

[5] Having set up the analytical tools, below, I examine five independent vignettes that demonstrate the breadth of music that engages with multiple pentatonic transformations on a structural level. I begin with transformations that complement form in a Cantopop song by Joseph Koo-Kar Fai and a character piece by Tan Dun. Then, I turn to the construction of mode cycles in Zhu Zaiyu's treatise and in the oral tradition of Liaoning shawm bands. Lastly, I discuss Bright Sheng's dissonant harmony generated by pentatonic transformations. Together, they paint a picture of transformations that unite music of vastly different types—historical and contemporary, regional and international, abstract and embodied, fantastical and practical, improvised and composed.

5. New Year Cantopop

[5.1] *Xuangong* transformations underpin one of the most famous Cantopop songs celebrating Lunar New Year, "Jukfuk nei" 祝福你 ("Bless you"), composed in 1980 by Joseph Koo Kar-Fai 顧嘉煇. Typical of Cantopop, the track combines the formal and harmonic structure of Western pop music with Cantonese instruments and melodic turns of phrase.

[5.2] "Jukfuk nei" is saturated with melodic *xuangong* transformations (**Example 5.1** and **Audio Example 7**). The intro and verse are sentential parallel periods with the same harmonic structure: I–V in the antecedent and V–I in the consequent. The melodic variation between antecedents and consequents closely follows the harmony. The melody (mm. 1–3) in the antecedent transposes by t_{-2} from its tonic form B♭, G, F to its dominant form F, D, C in the consequent. The dominant-form cadence in the antecedent transposes by the inverse t_{-3} to its tonic form in the consequent. With minor deviations, the bulk of the verse also transposes by t_{-2} from its antecedent to consequent. In general, transpositions in the intro and verse closely follow (diatonic) harmonic functions.

[5.3] In comparison to the intro and verse, the bridge contains a larger variety of transformations that suggest a deeper motivic network. **Example 5.2** connects the transformations in a single motivic network based on the opening "jukfuk nei" hook (B♭, G, F). In this network, the intro and verse only switch between tonic (**gong-yu-zhi**) and dominant forms (**zhi-jue-shang**) of the hook in B♭ pent. The bridge, in comparison, moves outside of B♭ pent. The q_1 going into the bridge (B♭, G, F → B♭, G, F) is pc-preserving, but both the pentatonic set and the positions change. Then, the new B♭, G, F in the bridge moves back to F, D, C via a position-preserving T_7 rather than the set-preserving t_3 heard earlier.[(22)] Towards the end of the bridge, the tonicized half cadence supports two overlapping motives: C, A, G at the melodic peak and A, G, F at the end. Unlike the q_1 earlier, the q_1 of A, G, F → B♭, G, F spotlights the A → B♭ leading-tone resolution back into the verse.

[5.4] Pentatonic transformations go hand in hand with form and harmony in "Jukfuk nei". Fifth-related $t_{\pm2}$ underpins the intro and verse, while the tonally unstable bridge features q_1, $T_{\pm7}$, and a rapid succession of t_1s. The enduring success of this song and the ubiquity of motivic transformations in it suggest it may be fruitful to explore the frequency and extent of these types of transformations in Cantopop.

6. A character piece

[6.1] The examples so far have been melody-centric. In Tan Dun's 譚盾 "Staccato Beans" from *Eight Memories in Watercolor* Op. 1 (1978–79), the most interesting *xuangong* transformations are in the harmony. The character piece for solo piano repeats a folk-like theme five times. Statements 1 and 3 of this theme have similar pentatonic Alberti-bass accompaniments, and phrases 2, 4, and 5 have a stepwise descending bass (**Example 6.1**). From phrase 1 to phrase 3, Tan transposes the melody and the final chord by a straightforward T_5, moving from F pent. to B♭ pent. These chords affirm the modal finals of D **yu** and G **yu**, respectively.

[6.2] In addition to T_5, *xuangong* transformations provide other ways to move from F pent. to B♭ pent., ones that Tan Dun takes advantage of in the accompaniment. Phrase 1 opens and closes on the same Dm^7 chord. Phrase 3, however, opens with $B♭^{add6}$, which temporarily shifts the modal center from G to B♭. The two chords, Dm^7 and $B♭^{add6}$, both with omitted thirds, may seem unrelated at first. But, for listeners immersed in Chinese pentatonic music, their q_n-equivalence is readily apparent. From phrase 1 to phrase 3, the opening chord moves by T_5t_1, adding an extra t_1 to the melody's T_5:

$$\text{DACA} \rightarrow (\text{GDFD}) \rightarrow \text{B♭FGF}.$$

[6.3] For phrase 2, Tan rearranges the Alberti bass in phrase 1 (D, A, C, A) into a half-note descent (D, C, A). In phrase 2, one can hear the full bass line as a t_0 repetition of the D, C, A descent, or as note-by-note transpositions t_{-1}, t_{-1}, t_2, t_{-1}, t_{-1}. The two are related; the cumulative t_0 is equal to the first three note-by-note transpositions:

$$t_0 = t_{-1}t_{-1}t_2.$$

In phrase 4, this equation takes on more significance. Phrase 4 begins with the transposed and syncopated descent GFD (which is also doubled at t_1 above), but it swaps the cumulative t_0 for a T_5, which momentarily brings the accompaniment into B♭ pent. Heard note-by-note, it forms an octave descent: t_{-1}, t_{-1}, $t_{-1}q_1$, t_{-1}, t_{-1}. The similar construction of phrases 2 and 4 suggests the dual interpretation

$$T_5 = t_{-1}t_{-1}t_{-1}q_1.$$

This sequence of events positions the last dyad at GB♭ so it resolves stepwise (diatonically) to A, the dominant of D **yu**. While the melody modulates via a straightforward T_5, *xuangong* transformations afford subtler relations in the harmony.

7. Confucian sequences

Xuangong pedagogy

[7.1] In his treatise *Lülü jingyi* 律呂精義 (Essence of tuning) (1596), Ming-dynasty prince Zhu Zaiyu 朱載堉 lavishly realizes *xuangong* theory through modal variations. While Zhu is known for the earliest calculation of twelve-tone equal temperament (xin fa mi lu 新法密率) (Rehding 2022, 260), his tuning theory was intertwined with modal theory through didactic qin etudes:

諸書雖載六十調八十四聲之說，然有體無用，而初學難曉。今以琴發明則體用兼備，庶初學易曉也。
(1596, seq. 447)

Although many texts describe the concept of 60 and 84 modes, they exist in theory only, not in practice, and they are difficult to understand for beginners. By contrast, the present use of the qin to expound modes combines theory and practice, so that modes may become easy to understand for beginners.

[7.2] In the treatise, Zhu realizes pentatonic transformations primarily through "Nanfeng ge" 南風歌 ("Song of the southern wind"), which spans over a hundred pages in *Lülü jingyi*.[23]

Southern wind's balm	南風之薰兮
can soothe people's anger.	可以解吾民之慍兮
Southern wind's timeliness	南風之時兮
can enrich people's wealth.	可以阜吾民之財兮

(Traditional, "Nanfeng ge" 南風歌)

According to legend, the song is composed by sage king Shun 舜 (ca. 2000 BCE). As Zhu points out:

> 舜彈五弦之琴，歌南風之詩，而天下治。 (Zhu 1596, seq. 546)
>
> Shun played the five-string qin, sang the poem of *Southern Winds*, and his realm was well governed.

Weaving music theory and the Confucian ideal of moral governance, Zhu illustrates pentatonic transformations using the poem and the qin. Mimicking the style of Confucian state-ceremonial music (Lam 1998; Woo 2017), Zhu set "Nanfeng ge" to a looping pentatonic scale (**Example 7.1**).(25) In antiquarian court music played by a full orchestra and chorus, each syllabic note is extremely slow (5 bpm), equal in length, and elaborated by figurations. Songs also have explicit modal organization and melodic patterns associated with them (Lam 1998, 130–31).(26) A simple scale would not be out of place in this style. To illustrate aspects of qin tuning, Zhu only employs open strings.

Straight as strung pearls

[7.3] In the *gongchepu* version of "Nanfeng ge", the diagonal invariants in Example 7.1 hint at internal symmetry. Six features characterize Zhu's setting:

1. The entire song consists only of descending steps (t_{-1}) in gongche notation:

   A 工 *gong* (unrelated to **gong** 宮)
   G 尺 *che*
   E 一 *yi*
   D 四 *si*
   C 合 *he*
   A 工 *gong*
   ⋮ ⋮ ⋮

   Zhu labels this stepwise descent as the *straight as strung pearls* pattern (*duan ru guanzhu ge* 端如貫珠格), and he relates this to the Daoist idea of non-interference:

   > 《樂記》曰:「大樂必易」，又曰「大樂與天地同和」，言其音調出於天生自然，不由人力編排，而累累乎端如貫珠，譬猶太羹玄酒，無味之中真味存焉。 (seq. 533)
   >
   > It is said in *Book of music*, 'great music is simple,' and 'great music is in harmony with heaven and earth.' They express how tones are *self-so* without human involvement, and so, when forming a sequence, tones in orderly succession are straight like strung pearls; like ceremonial sauce and wine, in no taste there exists true taste.(27)

2. Each song begins and ends on the modal final.
3. Features 1 and 2 above constrain the poem's length to $5n + 1$ syllables. "Nanfeng ge" has (5·5) + 1 = 26 syllables.
4. The 5 + 8 = 13 poetic meter (dashed lines in Example 7.1) articulates a full t_2 cycle (A, D, G, C, E, A), since $(t_{-1})^{13} = t_2$. The cycle contains three P4s and one M3.
5. Each column transposes the song by t_{-1}; five columns complete a modal cycle within the same set.
6. Features 1 and 5 explain the diagonal invariants. The score is a two-dimensional graph with the syllable-level t_{-1} on the y axis and the song-level t_{-1} on the x axis (**Example 7.2**). The diagonal invariants (t_0) are the difference of the two axes.

As the six features show, Zhu enriches an otherwise ordinary t_{-1} with Daoist philosophy, melodic schemata, hierarchical sequences, modal cycles, and visual appeal.

Two ge 格

[7.4] In addition to the *gongchepu* in Example 7.1, Zhu iterates "Nanfeng ge" through all 60 modes in fully figurated *qinpu* (tablature) and *lülüpu*. The lülüpu score, based on the first character of each lülü, contains two different 60-mode cyclic patterns (*ge* 格) (Examples 7.2 and **7.3**), and Zhu describes them like so:

右自南呂為羽，至於應鍾為宮，凡六十調，其名曰端如貫珠格。復自黃鍾之宮，至於林鍾之角，凡六十曲，其名曰循環無端格。此二格者，譬如先天八卦，橫圖縱圖，圓圖方圖，皆出於自然，非由人力也，學者宜詳味焉。中有難言之趣，當自得之耳。 (seq. 546)

From *nanlü* as **yu** to *yingzhong* as **gong**, altogether 60 tones, they follow the pattern *straight as strung pearls*. From **gong** of *huangzhong* to **jue** of *linzhong*, altogether 60 tunes, they follow the pattern *endless cycle*. These two patterns—like the primordial eight trigrams, the horizontal and vertical diagrams, and the circular and square diagrams—are *self-so* without human involvement; scholars should investigate them in detail. Therein is indescribable delight; one must grasp it themselves.

At the deepest hierarchical level, the two modulation patterns are metaphysical sequences connected to the Yi jing (I Ching) 易經. The modulating straight as strung pearls pattern doubly generates 60 modes by T_1 and t_{-1}. Example 7.2 shows the first spread of the mode cycle in *lülüpu*, of which the right side is identical to the *gonchepu* (Example 7.1), including the invariant diagonals. Each column transposes the song by t_{-1}, and each page transposes the five columns by T_1. The first page (right side of Example 7.2) shows "Nanfeng ge" in the five modes of *huangzhong* (C) pent., and the second page shows the five modes of *dalu* (C♯) pent.

[7.5] Elsewhere in *Lülü jingyi*, the *endless cycle* pattern (*xunhuan wuduan ge* 循環無端格) describes the equal-tempered *lülü* as a tone wheel, but as mode cycles, the term specifically describes the 60 modes singularly generated by q_1. **Example 7.3** shows the first two pages of the *endless cycle* pattern, and it highlights the changing note in each incipit. Each column transposes the song by q_1, and each page transposes the five-cycle by $q_5 = T_1$.[(28)] The two cycles showcase the basic transformations t_n, T_n, and q_n. The *straight as strung pearls* pattern mirrors the pentatonic and chromatic rings of the *xuangong* tone wheel, and, like Hook's signature transformations, the *endless cycle* pattern reveals the singular *xuangong* generator. Furthermore, Zhu fortifies pentatonic relations through rhetorical parallelism (*dui-ou* 對偶). He uses synonyms *qu* 曲 and *diao* 調 to refer to modal variants ("tune" and "tone" in my translation), and he pairs *weidiao* with the *straight as strung pearls* pattern (Example 7.2), and *zhidiao* with the *endless cycle* pattern (Example 7.3).

Qin tuning

[7.6] In "Nanfeng ge", each change in pentatonic set necessitates retuning the qin.[(29)] In *Lülü jingyi* (seq. 450), Zhu expands common qin tunings (Example 3.5) to all 12 pentatonic sets (Example 7.4). In his diagrams, he orders qin tunings by T_1 of the underlying pc_{pent}, but they are actually generated by $b_{\pm 1}$ to maintain string tension, so each string varies by at most two semitones. **Example 7.4** reorders Zhu's tunings with b_1 and T_1 as the two axes.[(30)]

[7.7] As seen in the excerpts above, Zhu's didactic ritual music is imbued with theoretical intent, and the degree to which Zhu luxuriates in transformations cannot be overstated. Confucian lyrics are set to music that recreates the legend of Shun's harmonious rule. Sequences permeate all levels of "Nanfeng ge", from syllable to poetic meter to mode cycles. In "combining theory and practice" (*ti yong jianbei* 體用兼備) (seq. 447), Zhu's Confucian sequences affirm the complementary theories of *xuangong* transformation and equal temperament, allowing the qin to play in all 60 modes.

8. Funeral *chaoyuan* cycles

[8.1] Section 2 (Example 2.4) showcased a *xuangong* variation technique by T_n in *guchuiyue*. One can only imagine how difficult it is for a listener to follow a five-mode t_n cycle (*wu diao chaoyuan* 五調朝元) or an expanded 35-mode cycle.[31] Countless instances of both cycles (*chaoyuan*) were meticulously transcribed in the volumes for Liaoning and Jilin province in the *Anthology of Chinese Folk Instrumental Music* 中國民族民間器樂曲集成 (Li et al. 1996, 2000). Often accelerating from extremely slow pulses—as slow as 3 bpm (Li et al. 1996, 61)—the cycles that were transcribed can easily last for over an hour (Yang 1996, 64–65). Transcriptions are plentiful, but unfortunately recordings remain rare, and the technique was thought to be extinct in the 2000s (Liu 2008, 28).

[8.2] Li Laizhang 李來璋 (1985; 1994) was one of the first to extensively theorize and transcribe these cycles. **Example 8.1** shows "Ku huangtian" 哭皇天 ("Cry for heaven") as it was played by Zhang Hanchen 張漢臣 from Liaoyuan, Jilin province (after Li 1985; 1994). **Example 8.2** shows "Ku huangtian" repeated in a five-mode cycle, with each modal variant of "Ku huangtian" abbreviated to its incipit and final. The cycle here uses a pivot note technique, such that the last note of one modal variant becomes the first note of the new one (Li 1985). In other words, the *chaoyuan* t_n is determined by the same t_n between a tune's first and last notes. Yet, as Li (1996, 63–65) and Lin (2011) demonstrate, the *chaoyuan* t_n is cognized by shawm players as a minimally perturbed T_n, such that

$$t_1 = T_2 q_2$$

$$t_2 = T_5 b_1$$

$$t_3 = T_7 q_1$$

$$t_4 = T_{10} b_2,$$

where, in general, $t_{2n} = q_{24n} = T_{5n} b_n$, and $-2 \le n \le 2$. For example, rather than thinking of t_2 as transposing up two steps, musicians found it easier to imagine the T_5 version and then apply b_1 to it. According to this method, the first three notes of "Ku huangtian" at t_2 would be cognized as E, D, C → (A, G, F) → A, G, E.

[8.3] Li (1994, 24) displays the thirty-five-mode cycle in a circular diagram (**Example 8.3**) using *jianpu*, which I have re-notated in staff notation (**Example 8.4**). To expand the five-mode *chaoyuan* to thirty-five modes, the last t_2 is composed with a b_1 to move to a new pentatonic set, forming the repeating block t_2, t_2, t_2, t_2, $t_2 b_1$. Although this sequence would eventually exhaust all 60 forms of "Ku huangtian", in practice, the diatonically tuned shawm can only fluently handle sets with up to three *qing* or three *bian* (see Example 3.4, line a above). After reaching the *bian*-most scale of A pent. in variation 20, players take a T_6 shortcut to the *qing*-most scale of E♭ pent. in variation 21. (Both variations 21 and 36 begin with a pseudo-pivot note a semitone apart, as indicated by asterisks in Example 8.3.) Then, the sequential blocks continue until they reach the original mode again.

9. *Xuangong* × Viennese trichord

[9.1] Contemporary Chinese composers have seamlessly blended pentatonicism with modern dissonant idioms, such as serialism (Rao 2002), and have likewise integrated *xuangong* transformations to great effect. Chinese-American composer Bright Sheng is well known for his dissonant treatment of folk song influenced by Béla Bartók (Sheng 1998; Wong 2007). "Tibetan Air," from *Three Fantasies* (2005) for violin and piano, features contrasting sets in the horizontal and vertical dimensions with the pentatonic set (02479) in the melody, and Viennese trichord (016) in the harmony. The violin and piano begin the movement with emphatic Viennese trichords and major sevenths (a subset of the trichord) in extreme registers. Then, with a dissonant accompaniment, the violin presents a long, spun-out pentatonic theme inspired by Tibetan folk song (Sheng n.d.).

[9.2] The entire movement is in ABA' form, and my analysis will focus on transformations of the opening melody in the first two sections. Examples 9.1 to 9.2 briefly cover the A section, and Examples 9.2 to 9.5 describe the B section in greater detail. Sheng's music has an irreducible complexity that pushes the current style of score annotation to its limit. From phrase to phrase, there are judicious additions and omissions of notes, lines with no clear segmentation, and seemingly familiar turns of phrase that are difficult to pinpoint. In the following examples, I have attempted to strike a balance between fidelity and clarity.

[9.3] Transformations of the opening theme are dominated by simple fifth-based transpositions (**Example 9.1**). The violin starts with a melody in F♯ pent. that continues with inexact repetitions. Two measures later, the piano picks up the theme again at T_7 at half the speed and with extra notes (mm. 10–13). At the climatic ending of the A section, the violin repeats the piano theme at b_1 with even more added notes. So, by the end of the A section, the theme has accrued a cumulative transposition of $T_7b_1 = b_{26}$ since it was first heard. The second half of the violin climax moves by T_7, an alternative to b_1 that arrives at the same pentatonic set (C♯ pent. → A♭ pent.).

[9.4] To link the A and B sections, the violin's hushed B section reworks its A-section material by liberally omitting notes and by subtle b_1 and b_2 transpositions (**Example 9.2**). The violin plays the two versions back-to-back, so the linkage is relatively easy to hear.

[9.5] The entire B section is a double parallel organum. The violin and piano play a high and fast two-voice organum (top of **Example 9.3**), and the piano also plays another low and slow three-voice organum (bottom of Example 9.3). (The lowest voice of the trio drops out mid-way.) The high and fast organum combines, in simultaneity, the violin parts that open and end the A section. Therefore, the opening parallel interval $T_7b_1 = b_{26}$ encapsulates the transformational journey heard across the entire A section (diagrammed above in Example 9.1).

[9.6] The high/fast organum continues into a *fortspinnung* (m. 48, top of Example 9.3)—to borrow a term from Baroque music—that develops the motive through various transformations. Horizontally, the melody transforms by T_5, then T_7. Vertically, the organum interval b_{26} teeters a bit, but it eventually settles on $b_{25} = T_7$. I will return to this general trajectory later. Below the high/fast organum, the piano plays contrabass chord changes whose meaning is opaque at first. Though obscured by the low register and enharmonic spelling, the left hand plays the low/slow organum (bottom of Example 9.3), with Viennese trichords harmonizing a melody traceable to the high/fast organum. The *fortspinnung* of the **jue-gong-shang** (or **mi-do-re**) tail motive (mm. 45–46), which is more apparent in the low/slow organum, confirms this similarity. As shown by its interval vector <100011>, the Viennese trichord has pc invariance only at $T_{\pm1}$, $T_{\pm5}$, and T_6. In mm. 48–50, the low/slow organum chords are voiced, from top to bottom, at T_6-then-T_5, which sums to T_{11}. In this way, the Viennese trichord governs the vertical dimension. Since harmony is generated from the melody down, and no functional bass is present, all my transformations start from the higher voice.

[9.7] Phrase 2 of the low/slow organum (mm. 51–53, bottom of Example 9.3) varies the voicing of the (016) trichord. This time, the voicing (from top to bottom) sums to T_1 rather than T_{11}. Rather than voicing the chords with T_6-then-T_7 or T_7-then-T_6, Sheng's composition yields b_{26}-then-b_{29}, transformations, which also sum to T_1 ($b_{26}b_{29} = b_{55} = T_1$) and preserve the Viennese trichord. At the level of individual pcs, b_{26}-then-b_{29} admits both T_6-then-T_7 and T_7-then-T_6 voicings. Compare, for example, the last two chords of phrase 2 (m. 53), which are, from top to bottom, G, C♯, A♭ and A, E, B♭. Unlike the high/fast organum, which takes only two measures to settle to T_7, the low/slow organum takes 16 measures (most of the B section) to settle to T_7.

[9.8] From the observations above, a script for the organum interval emerges: it moves gradually from dissonance to consonance using the b_n continuum from $T_6 = b_{30}$ to $T_7 = b_{25}$, which contain pc

intervals that originate from (016). The table in **Example 9.4** illustrates this process in greater detail. From b_{30} to b_{25}, for each b_{-1} increment, one consonant T_7 replaces one dissonant T_6 until all intervals are consonant. In effect, *xuangong* transformations provide Sheng a finer gradation of transpositions between T_6 and T_7. The violin's trajectory in the A section foreshadows this complex relation through the composition of fifth transpositions T_7 and b_1. While expressions such as b_{29}, T_7b_4, and T_6q_1 may be mathematically equivalent, they suggest different experiential pathways. It is possible to hear the transformations all three ways, even as 29 successive *bian*s. If the pianist playing Bright Sheng's music had also studied Li Yinghai's piano etudes (Example 3.7), they could be intimately familiar with the full cycle of 59 successive *qing*s, and hear the 29 successive *bian*s in that way.

[9.9] As the low/slow organum completes its dissonant-to-consonant trajectory spanning the entire B section, the high/fast organum completes this trajectory three times. The density of the organum forbids a complete representation here; **Example 9.5** shows the tail end of the three trajectories (phrase 1, phrase 4, and phrases 6–8), where most of the action takes place. (Other phrases hover around b_{27} without resolving to $b_{25} = T_7$.) The large dashed boxes in Example 9.5 show that the top voices of these phrases relate by some combination of fifth transposition. The top voice of phrase 4 is a verbatim T_5 of phrase 1. And phrase 6 begins at $b_{26} = T_7b_1$ of phrase 1 (see Example 9.1 above), the same transformation governing the A-section violin trajectory and the opening organum interval.

[9.10] Each of the three trajectories in the high/fast organum have their own character (Example 9.5), but their T_n endings ensure both organum voices end with the **jue-gong-shang** tail motive. As discussed, Phrase 1 teeters around b_{26} before resolving to $b_{25} = T_7$. Phrase 4 remains steadfast on b_{26}, and it resolves to $b_{25} = T_7$ only at the last minute. And, as a group, phrases 6–8 present the full, gradual shift from total dissonance to total consonance. Phrases 6–8 start at an easily audible $b_{30} = T_6$, overshoots to b_{31}, and gradually resolves to $b_{25} = T_7$. The last stretch of this gradual shift (mm. 61–62) takes place over an extended *fortspinnung* that lengthens the tail motive by melodic transformations $T_5b_{\pm1}$ (bottom right of Example 9.5). The organum interval remains consonant until the A section returns as a stretto canon (m. 68, not shown).

[9.11] The networks above demonstrate the following:

1. Example 9.1: how the theme transposes by T_7 and then b_1 across the A section
2. Example 9.2: how the theme at T_7b_1 is reworked for the B section
3. Example 9.3: how the B section is structured as a double parallel organum, with the high/fast organum proceeding at a parallel $T_7b_1 = b_{26}$ and the low/slow organum voiced as a Viennese trichord
4. Example 9.4: how the organum interval gradually shifts from the dissonant $b_{30} = T_6$ to the consonant $b_{25} = T_7$

The melodic dimension is defined by the pentatonic set, and the harmonic dimension is defined by the Viennese trichord, particularly its internal transpositions T_6 and T_7. The central transformation T_7b_1, containing both T_7 and T_6 of individual pcs, neatly captures the dynamic balance of dissonance and consonance.

10. Conclusion and my canons

[10.1] *Xuangong* transformation is subtly complex. For some, it guarantees "indescribable delight" (Zhu 1596, seq. 546), and for others, it creates unparalleled confusion (Pian 1967, 43; Liu 2005, 102). The goal of the present theory is to make the delights more describable and to clarify potential

ambiguity, and it does so by providing precise language for pentatonic relations, and by connecting a rich variety of existing theories and analytical methods, old and new. The eclectic range of examples show the extent to which these transformations permeate Chinese culture, from New Year pop songs to rural funerals, from concert halls to court rituals, and from ancient cosmology to modern-day counterpoint. Similar 5-in-12 pentatonic structures exist in French impressionism, jazz improvisation (Ricker 1983), and guitar pedagogy (Kolb 2014), all of which are rich avenues for further study. I also write in the hope that *xuangong* may inspire new musical compositions.

[10.2] My own musical offering, *Secret Symmetries*, movements I–V (2017) (**Example 10.1**), extend Duan Pingtai's t_n canon to a set of $t_n b_2$ canons at all five *n*s. My canons superimpose two pentatonic sets a M2 apart, whose union is the diatonic set (**Example 10.2a**). Movements II and III are privileged in that they consist of straightforward transformations $t_0 b_2$ and $t_1 b_2 = T_2$, respectively. The central motive is a pentatonic step that varies in its intervallic size (m3/M2) and combinations of these sizes (**Example 10.2b**). For example, the $t_4 b_2$ of mvt. I transforms the cello's opening M2 into the violin's m3. By comparison, in mvt. III, both parts begin with M2 and m3 in succession, and the T_2 imitation ensures that the imitation never changes the specific interval size. The quarter-note delay further obscures the canon, encouraging repeated and engaged listening to fully hear the *xuangong* relations within.

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Footnotes

1. All terms are romanized using Mandarin Hanyu Pinyin. The exceptions are Cantopop song titles, which are represented using Yale Romanization. Unless otherwise noted, all translations are my own.
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2. "Choiwan jeui yyut" is also known as "Jidu hua luo shi" 幾度花落時 ("When the flowers fall") and "Hong chen" 紅塵 ("Fleeting world") in popular music.
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3. In some styles, like *yifan* mode, *fan* is tuned sharper and *yi* lower.
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4. I have adapted the ordered-pair nomenclature I introduced for diatonic pitch classes in Lam 2020.
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5. Pentatonic positions here are analogous to diatonic positions in Lam (2019, 2020).
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6. The characters for *xuan*, 還 and 旋, are interchangeable in this context. "Six *lü*" refers to the whole-tone scale.
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7. I follow Jones (2017) in distinguishing the term shawm and *suona* for similar instruments. The former refers to the instrument rooted in oral traditions and traditional ceremonies, where it has different regional names such as *weirwa*, *wazi*, or *laba*. The latter is more commonly associated with conservatories and urbanized settings (Jones 2017).
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8. *Shuilong yin* is the only *qupai* that the Hua family band transpose regularly.
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9. *Gong* 工 in *gongchepu* (diatonic **mi**) and **gong** 宮 as a pentatonic position are unrelated.
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10. See Fan (2013, 117–27), for example.
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11. The *bian*- and *qing*-directed transformations can be heard as repeated voice-leading schemas (Tymoczko 2020, 119; 2005).
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12. *Gong* 工 in *gongchepu* (diatonic **mi**) and **gong** 宮 as a pentatonic position are unrelated. Readers of the present journal may be familiar with Chen Yi's atonal variation of "Baban" (Roeder 2020).
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13. The homonyms *fan* 反 in *fanxian* and *fan* 凡 in *bianfan* are unrelated. Contrary to *bian* 變 in *biangong* 變宮, which is the lowering prefix (definition 4), *bian* 變 in *bianfan* 變凡 means "to change" (definition 1 in section 3.3), and it raises fan by semitone. See Du and Qin (2007, 200–233) for further discussion of terminology.
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14. Shawm playing varies from region to region, and so does its terminology (Jones 2007), so these terms are not universal.
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15. The *gongche* syllable *shang* is unrelated to the pentatonic position **shang**.
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16. *Yigong* 移宮 (moving **gong**) is another term for *xuangong*.
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17. "一、養成用鄰指彈奏小三度'五聲式級進'的習慣,使隔指及鄰指都能應用。 二、熟悉鍵盤上各調及各調式的音階及一般音調" (Li 2002, i).
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18. Any two of the pentatonic elements pc_{final}, pc_{pent}, pos_{final} imply the remaining third.
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19. In Pian 1967, *zhidiao weidiao* appears as *jy diaw wei diaw* in the short-lived *Gwoyeu Romatzyh* romanization. *Song* appears as *Sonq*.
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20. Liu 2013 compares different perspectives on the history of terms relating to *xuangong*, including prominent interpretations by Huang (1993) and Chen (2004).
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21. For example, rotating the central arrow clockwise is the same as rotating the two rings counterclockwise, the directions "left" and "right" are ambiguous on the clockface, the relative direction might change if *huangzhong* and *gong* are placed on top or viewed upside-down.
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22. In a network similar to Examples 1.2 and 3.10, in Example 5.2, the arrows connecting the verse and the bridge show that $t_3T_5 = q_1$.
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23. Note the perfect rhythm achieved through the exclamatory article *xi* 兮 that ends each line.
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24. Punctuation mine. Zhu is citing the "Book of music" in *Records of the grand historian* (史記-樂書) by Sima Qian 司馬遷 (ca. 100 BCE). Sections of it are nearly identical to the "Book of music" in *Book of rites* (禮記-樂記). The poem "Nanfeng ge" is purportedly authored by Shun, and it has inspired numerous musical works (Thompson n.d.b).
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25. See Woo 2017 (128) for the full score.
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26. Woo (2017, 154) estimates the tempo to be 5 bpm, or 12 seconds per beat, based on the prescribed choreography.
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27. Ziran 自然 here means "self-so" in the Daoist sense (Wang 2020, 233–41). Its meaning in modern Chinese, "natural," is not exactly the same.
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28. Below each variation in Example 7.3, Zhu describes each semitone change as "*bian*," from "one *bian*" up to "five *bian*", that links one page to the next. Here, *bian* refers to the more general meaning, change, and each *bian* represents a q_1 rather than b_1.

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29. For a detailed discussion of Zhu's qin tuning and related aspects of *xuangong*, see Guo (1993) and Ding (1987). Zhu's *huangzhong* is tuned near E2 (Guo 1993).
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30. See Woo (2017, 261ff.) for additional commentary. *Lülü jingyi* Inner Book 7 continues with diatonic qin tuning.
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31. A seven-mode diatonic cycle also exists (Yang 1996, 65).
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32. Lin 2011, accessible online at https://musicology.cn/papers/papers_7129.html, provides transcribed examples for each t_n.

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