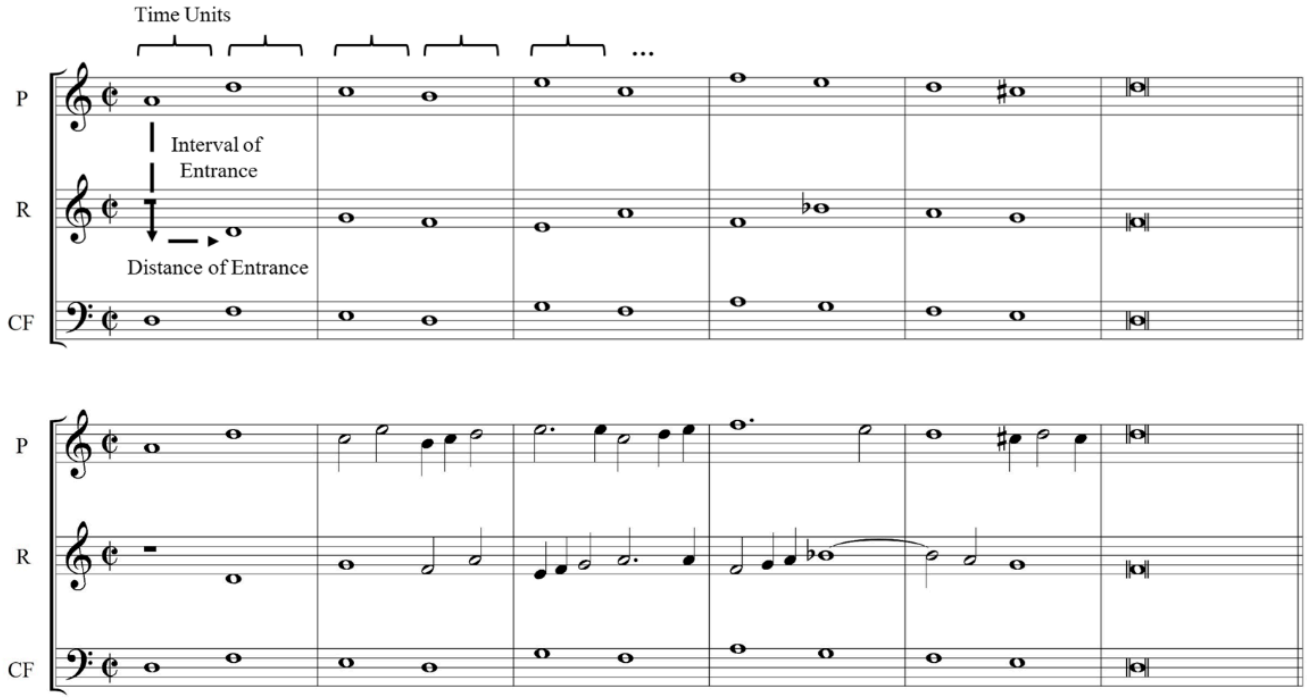
A JOURNAL OF THE SOCIETY FOR MUSIC THEORY

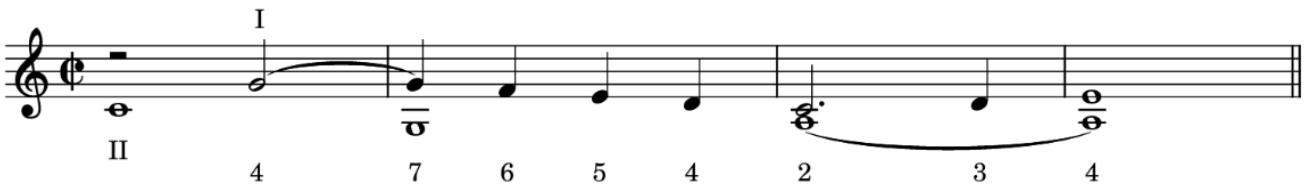
MTO 30.2 Examples: Gran, A General Method For Composing a Canon Against a Cantus Firmus Using Sergei Taneev's Double-Shifting Counterpoint

(Note: audio, video, and other interactive examples are only available online)
https://mtosmt.org/issues/mto.24.30.2/mto.24.30.2.gran.html

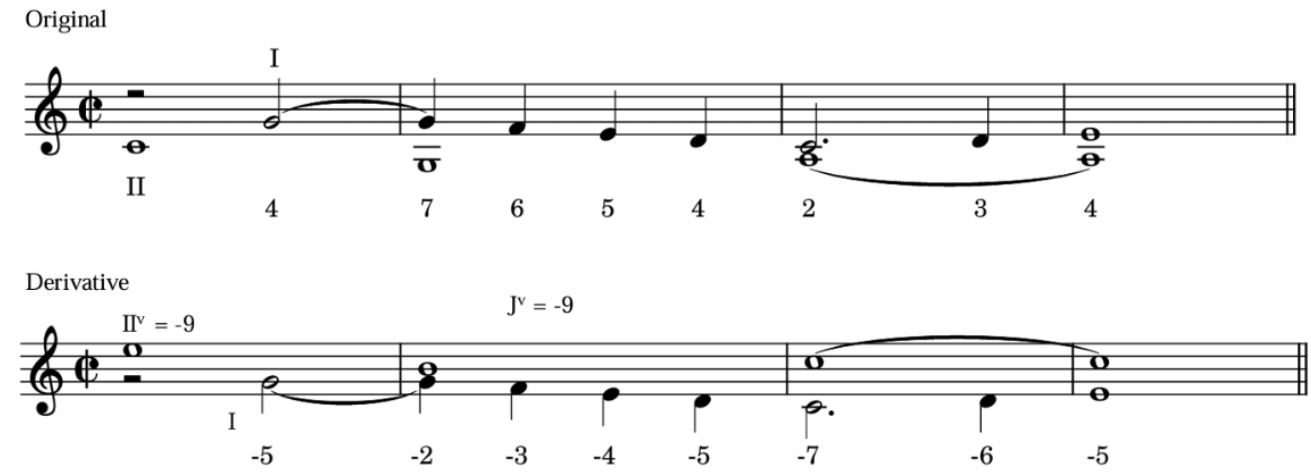
Example 1. A whole-note framework for a canon at the lower fifth at a distance of one whole note by Denis Collins set against Fux's Dorian cantus firmus, and Collins's florid realization of the same canon (Note: all examples are realized with MIDI unless otherwise indicated)



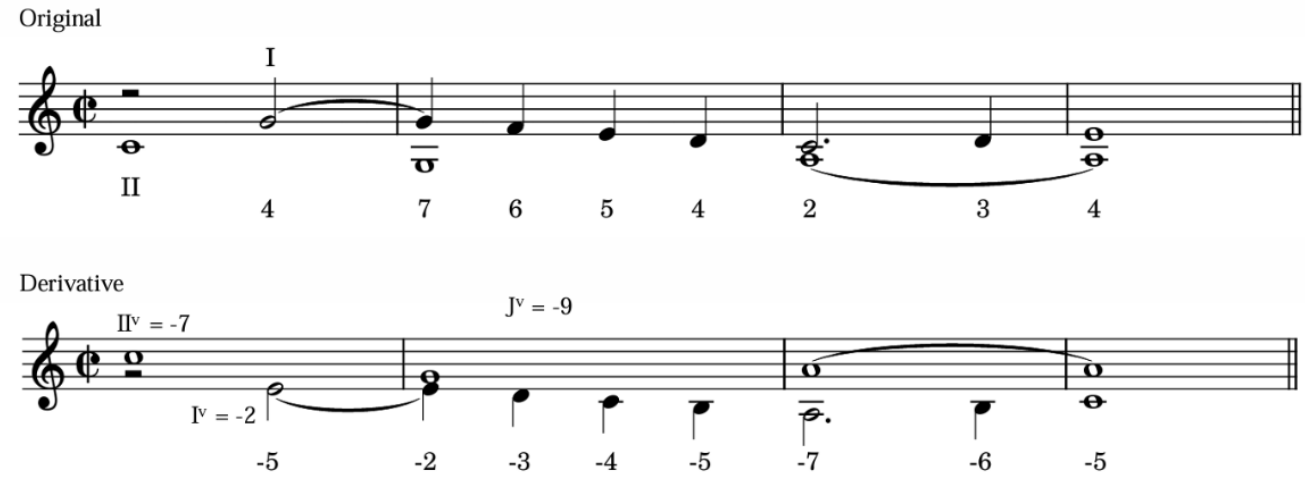
Example 2. An example by Taneev of a two-voice original combination



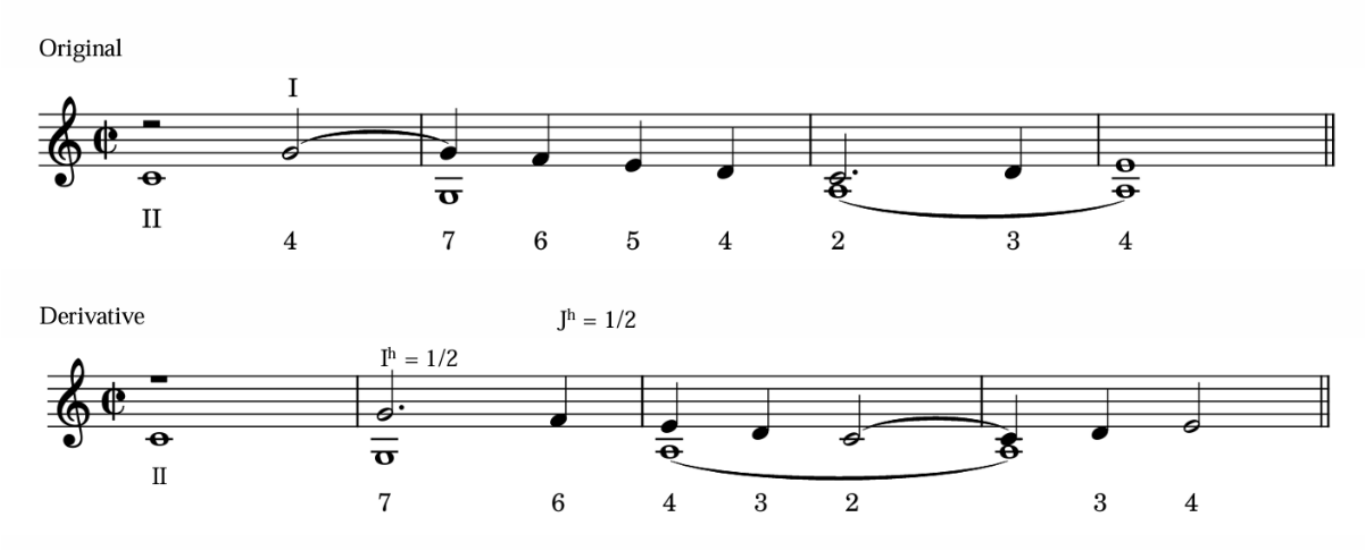
Example 3. A derivative combination brought about by a vertical shift of the original



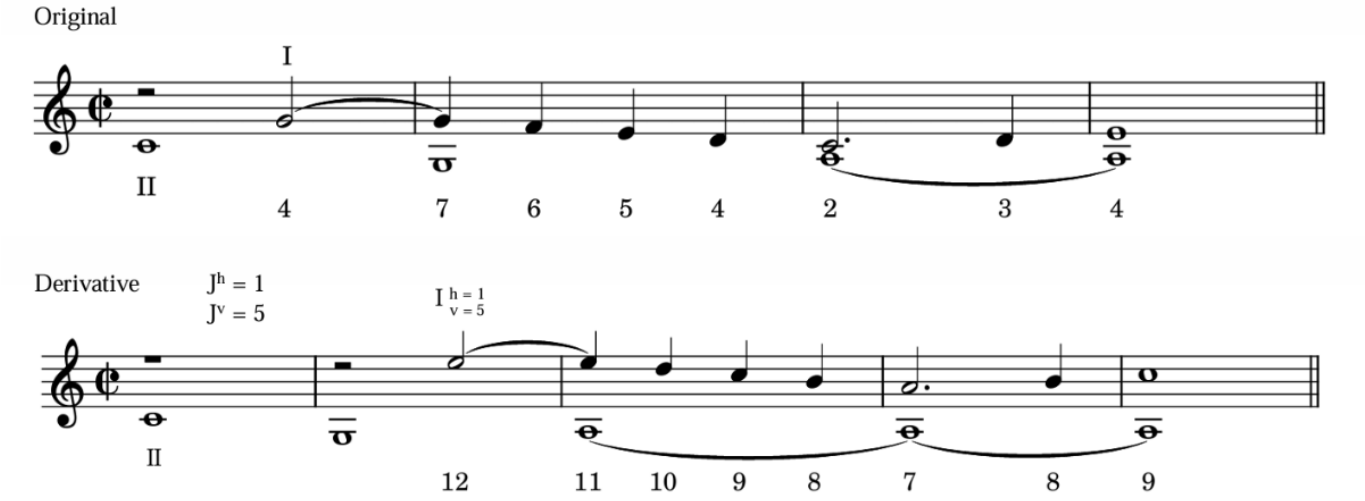
Example 4. A derivative where both voices have been vertically shifted and the vertical-shift index is the sum of the shifts



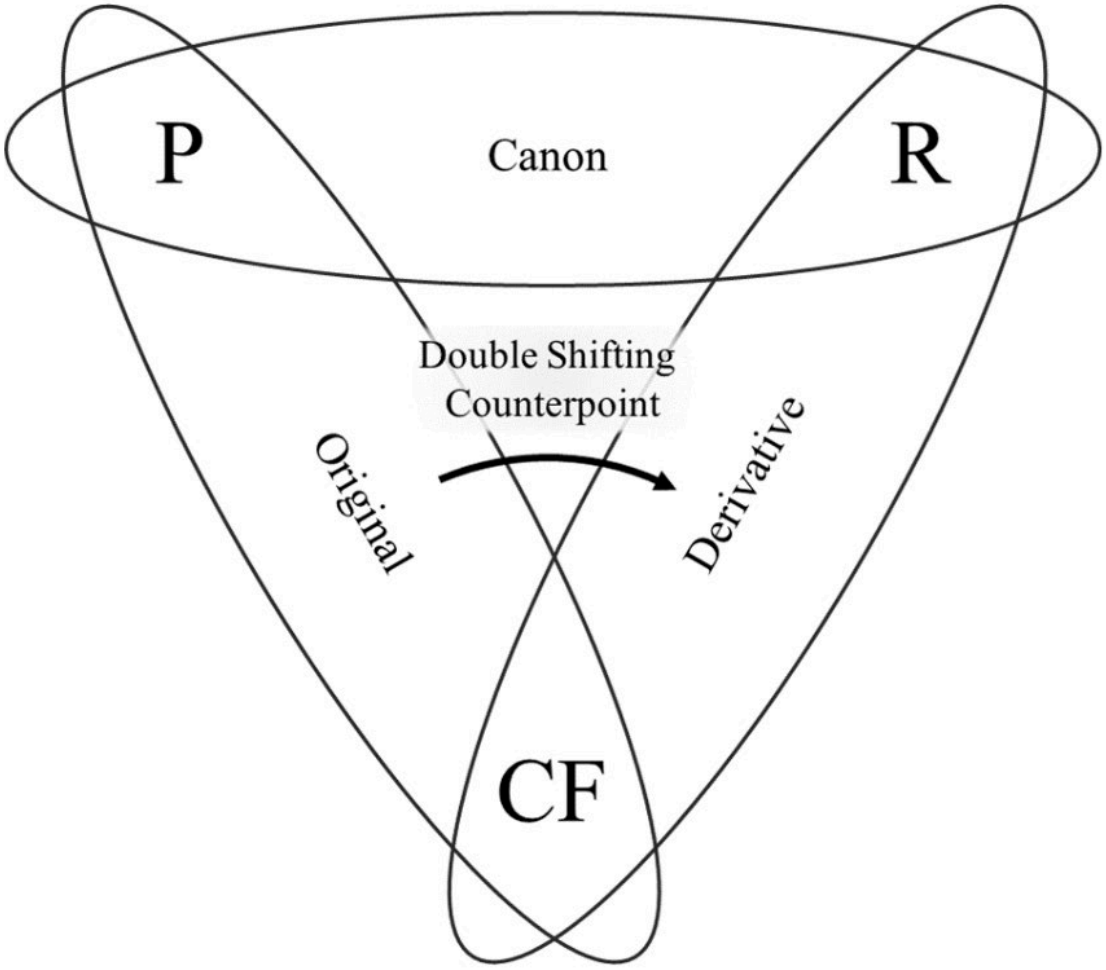
Example 5. A derivative combination brought about by a horizontal shift



Example 6. A derivative combination brought about by a double shift



Example 7. A summary of the basic problem in terms of double-shifting counterpoint



Example 8. An inversion table for invertible counterpoint at the twelfth (Jv = −11) illustrating fixed consonances and variable intervals

Original:	0	1	2	3	4	5	6	7	8	9	10	11
Derivative:	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0
Original:	unison	2nd	3rd	4th	5th	6th	7th	8ve	9th	10th	11th	12th
Derivative:	12th	11th	10th	9th	8ve	7th	6th	5th	4th	3rd	2nd	unison

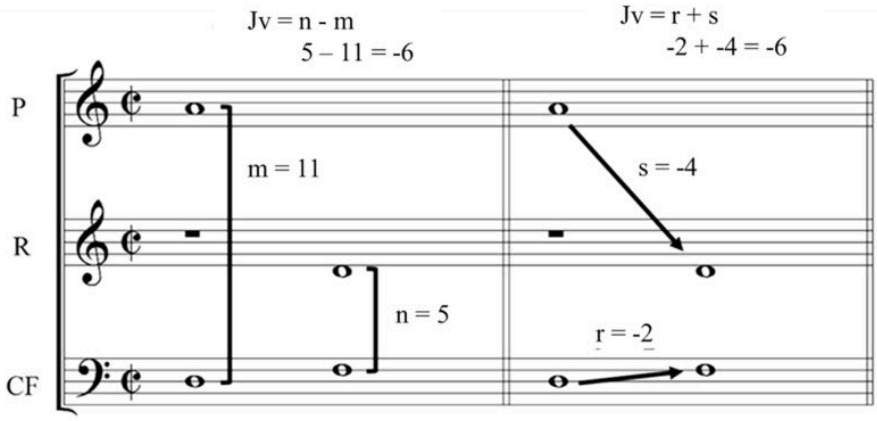
Example 9. An inversion table for invertible counterpoint at the fourth (Jv = −3) illustrating inverse and direct vertical-shifting counterpoint

	Inverse Shift				Direct Shift							
Original:	0	1	2	3	4	5	6	7	8	9	10	11
Derivative:	-3	-2	-1	0	1	2	3	4	5	6	7	8
Original:	unison	2nd	3rd	4th	5th	6th	7th	8ve	9th	10th	11th	12th
Derivative:	4th	3rd	2nd	unison	2nd	3rd	4th	5th	6th	7th	8ve	9th

Example 10. A table of the permitted intervals between proposta and cantus firmus which Collins (2008, 119) compiled using the Renaissance approach for a canon at the distance of one whole note with the cantus firmus in the lowest voice

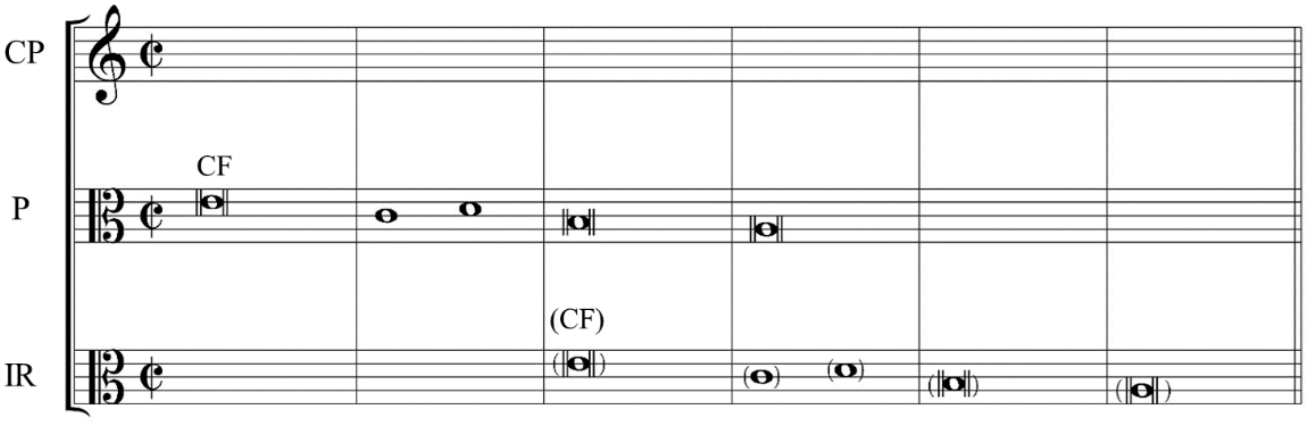
CF Melodic Intervals	Interval of Entrance (row 2) Permitted harmonic intervals between CF and P (rows 3 and following)						
	Unison or ↑↓8ve	↑2nd or ↓7th	↑3rd or ↓6th	↑4th or ↓5th	↑5th or ↓4th	↑6th or ↓3rd	↑7th or ↓2nd
Unison or ↑↓8ve	3rd 5th 6th 8ve	5th	3rd 6th 8ve	3rd 5th	6th 8ve	3rd 5th 8ve	6th
↑2nd or ↓7th	6th	3rd 5th 6th 8ve	5th	3rd 6th 8ve	3rd 5th 8ve	6th 8ve	3rd 5th 8ve
↑3rd or ↓6th	3rd 5th 8ve	6th	3rd 5th 6th 8ve	5th	3rd 6th 8ve	3rd 5th	6th 8ve
↑4th or ↓5th	6th 8ve	3rd 5th 8ve	6th	3rd 5th 6th 8ve	5th	3rd 6th 8ve	3rd 5th
↑5th or ↓4th	3rd 5th	6th 8ve	3rd 5th 8ve	6th	3rd 5th 6th 8ve	5th	3rd 6th 8ve
↑6th or ↓3rd	3rd 6th 8ve	3rd 5th	6th 8ve	3rd 5th 8ve	6th	3rd 5th 6th 8ve	5th
↑7th or ↓2nd	5th	3rd 6th 8ve	3rd 5th	6th 8ve	3rd 5th 8ve	6th	3rd 5th 6th 8ve

Example 11. The first two time units of Collins's canon from Example 1, illustrating how equation (3) and equation (4) can be used to calculate the vertical-shift index

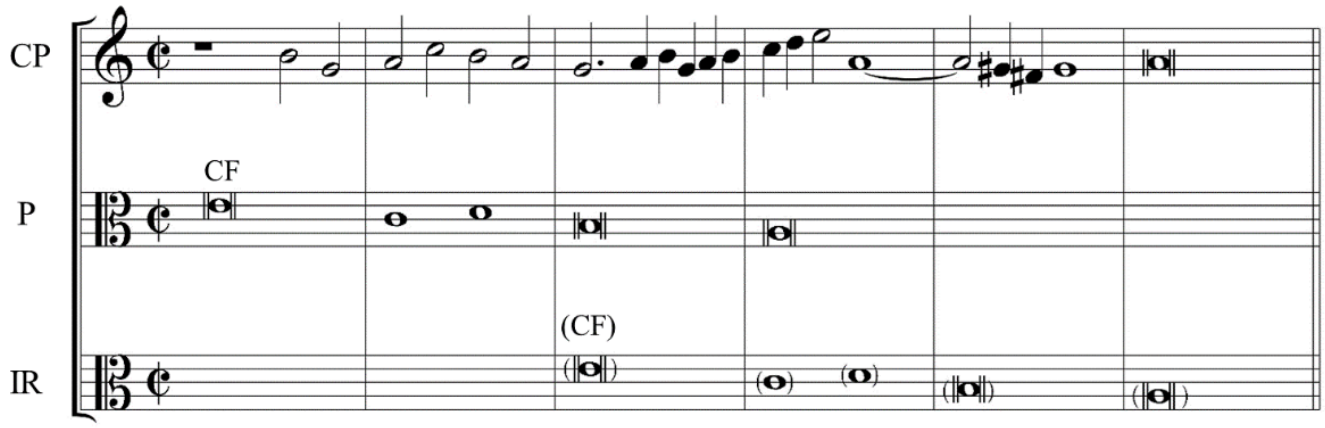


Original:	0	1	2	3	4	5	6	7	8	9	10	11	12
Derivative:	-6	-5	-4	-3	-2	-1	0	1	2	3	4	5	6
Original:	unison	2nd	3rd	4th	5th	6th	7th	8ve	9th	10th	11th	12th	13th
Derivative:	7th	6th	5th	4th	3rd	2nd	unison	2nd	3rd	4th	5th	6th	7th

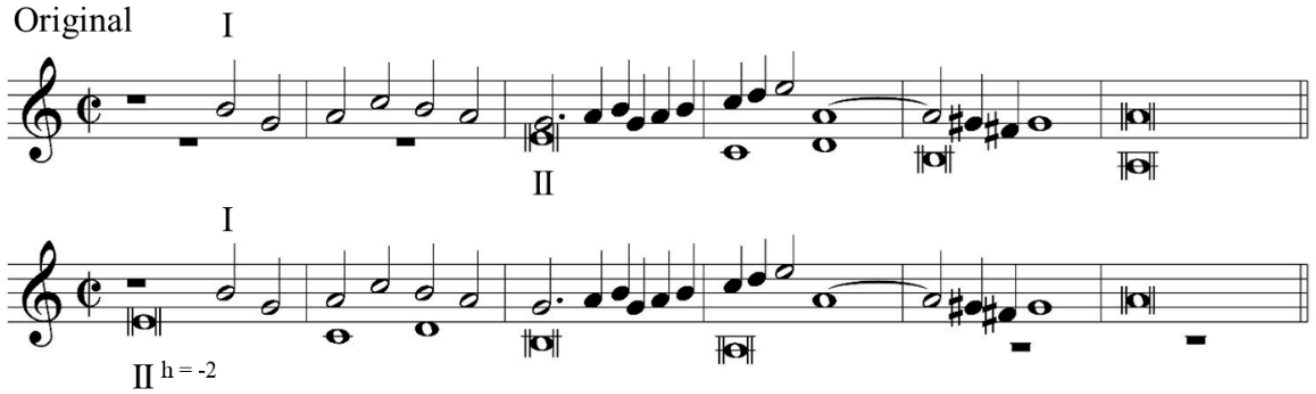
Example 12. The setup for composing a "basic version," wherein a cantus firmus is placed in canonic imitation at the unison with itself



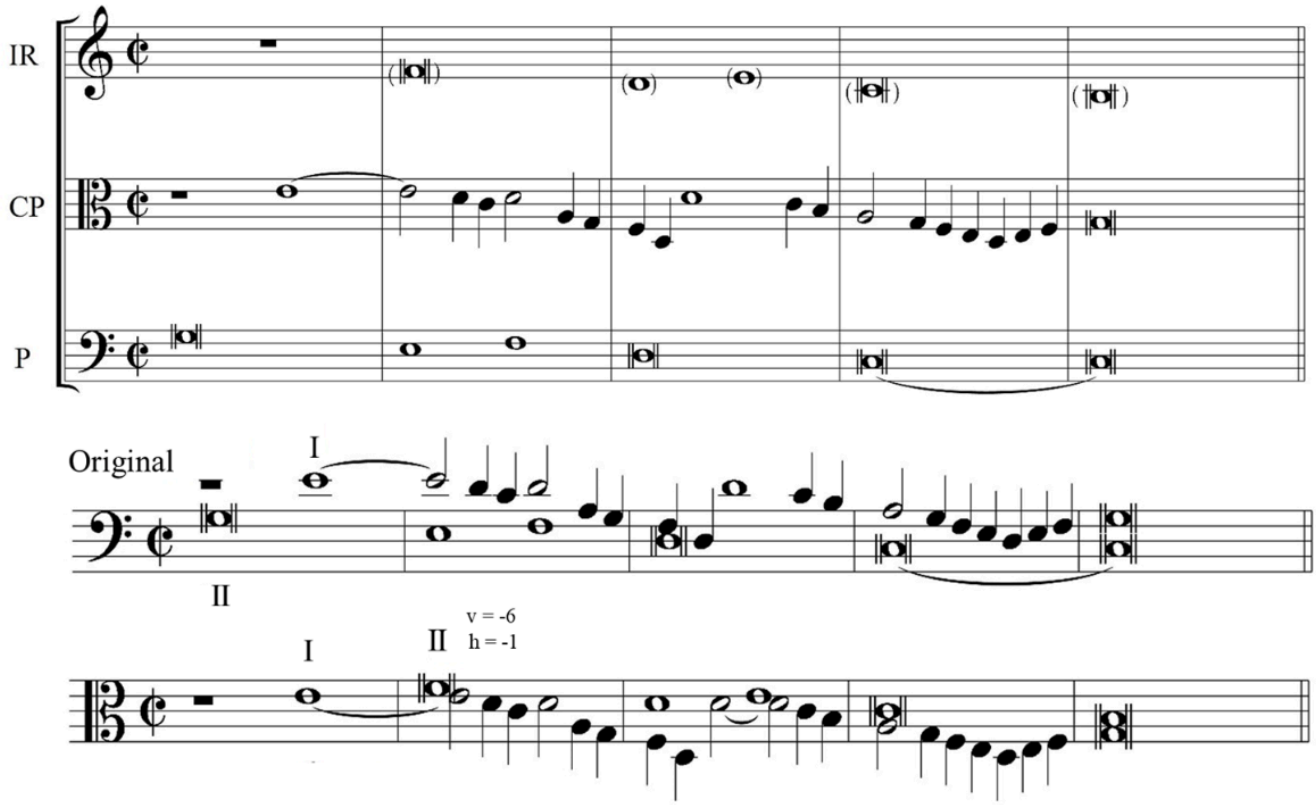
Example 13. A counterpoint composed to form correct two-voice combinations with either the P or IR forms of the cantus firmus independently



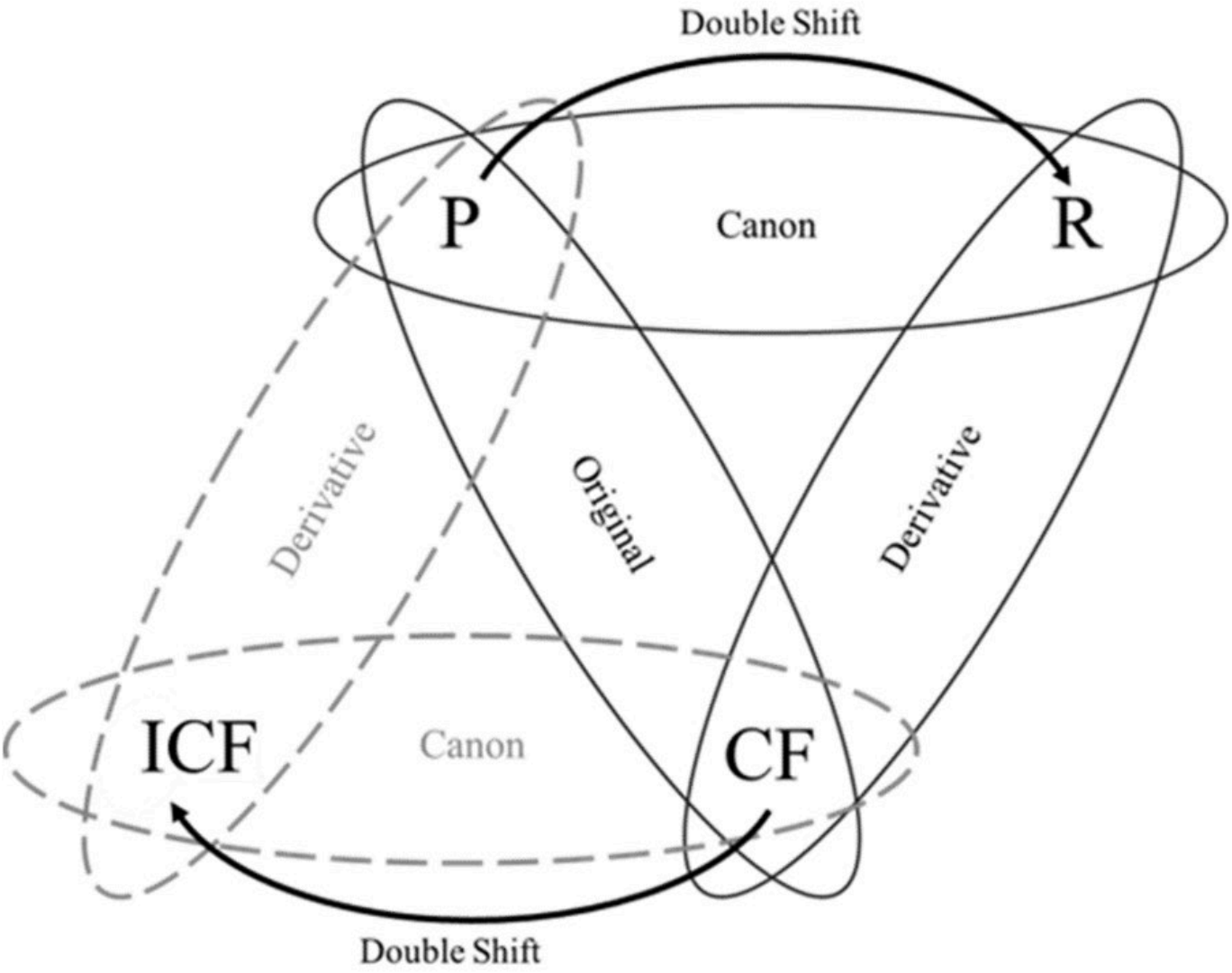
Example 14. Original and derivative combinations related by horizontal-shifting counterpoint formed from two-voice extracts from the basic version



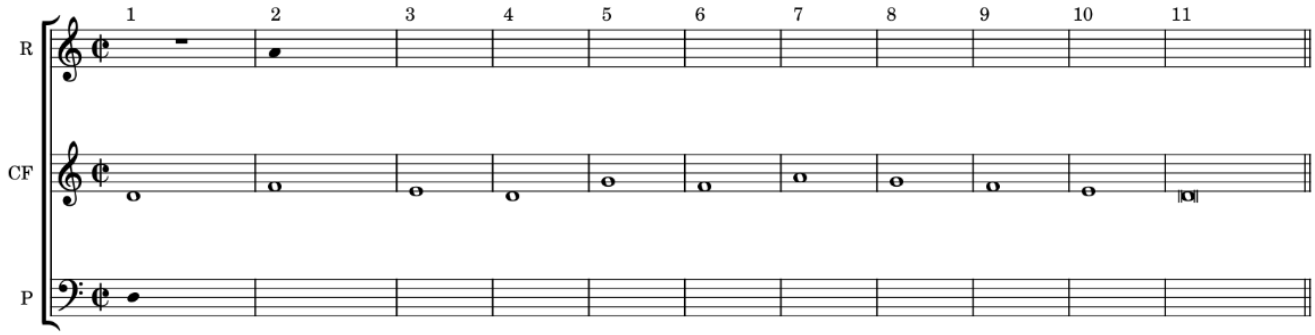
Example 15. A basic version wherein the canonic imitation of the cantus firmus is at an interval other than the unison and the resulting original and derivative combinations are related by double-shifting counterpoint



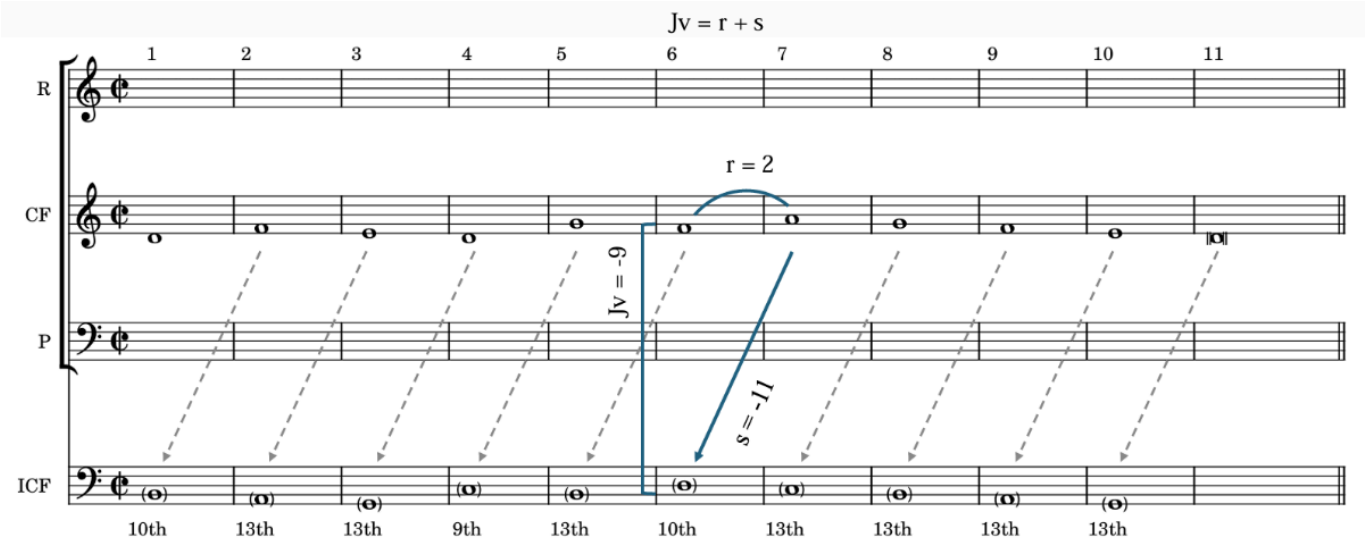
Example 16. The new canonic relationships introduced by the imaginary cantus firmus



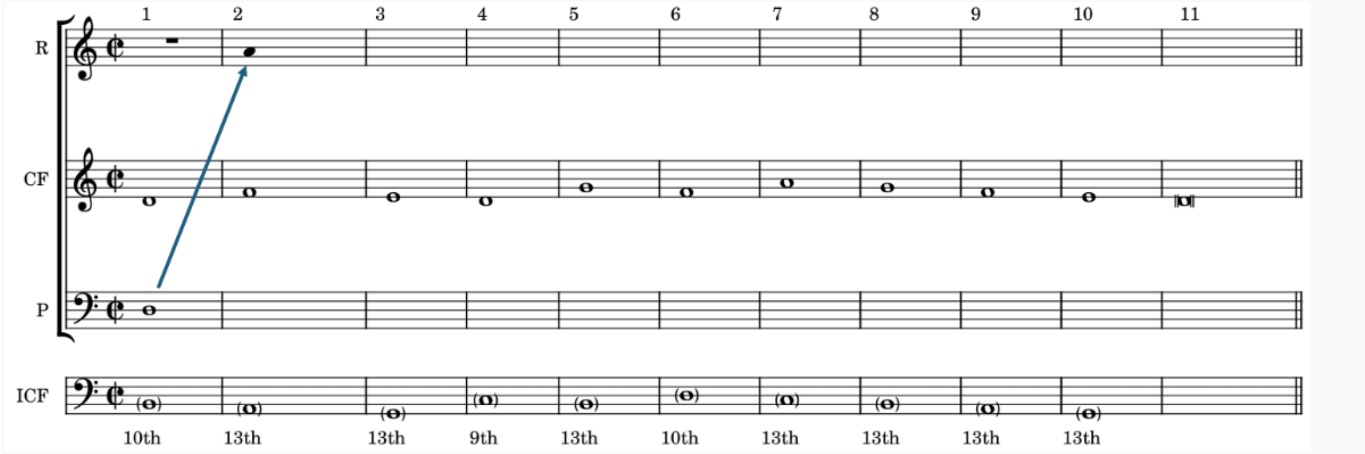
Example 17. The setup for a canon at the upper twelfth at a distance of one whole note with the proposta in the bass, Fux's Dorian cantus firmus in the alto, and risposta in the soprano



Example 18. Finding the placement of the imaginary cantus firmus

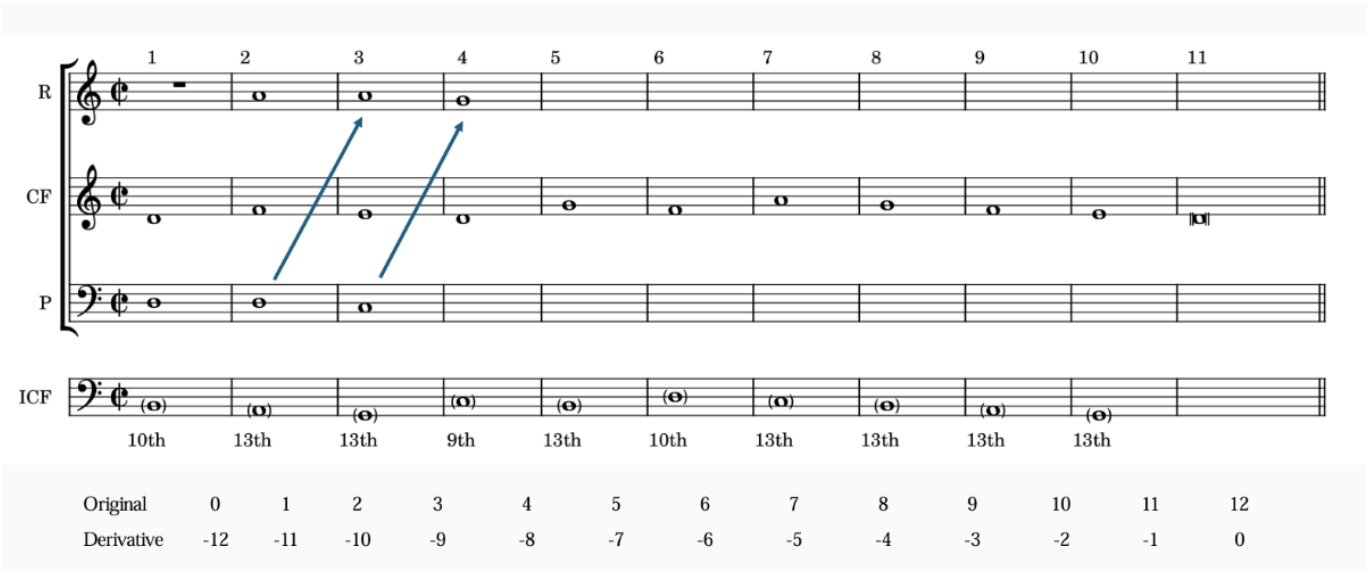


Example 19. Using the guide intervals to select notes for the proposta within a given time unit and the inversion table for invertible counterpoint at the tenth



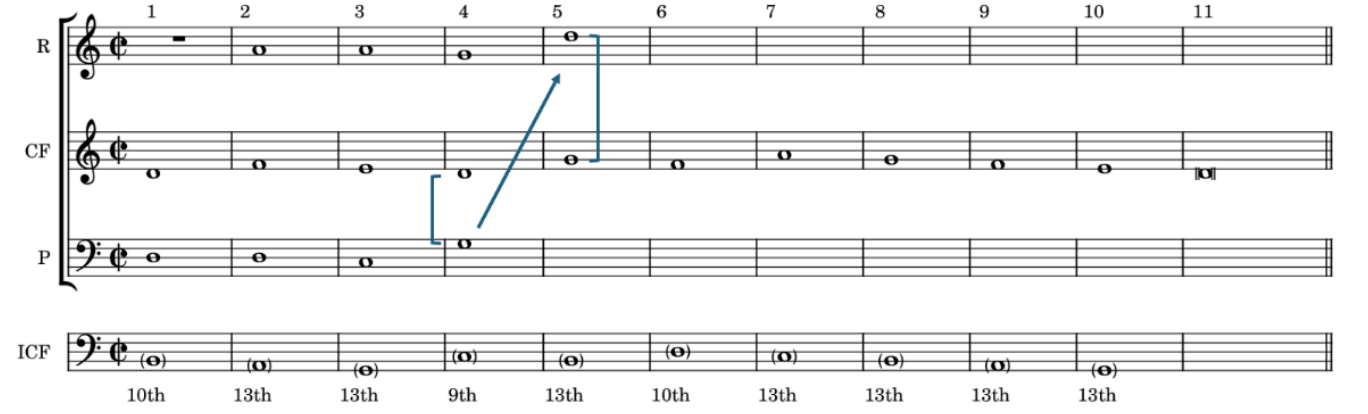
Original	0	1	2	3	4	5	6	7	8	9
Derivative	-9	-8	-7	-6	-5	-4	-3	-2	-1	0

Example 20. Selecting a note for the proposta in a time unit given the obligations of the risposta, cantus firmus, and imaginary cantus firmus



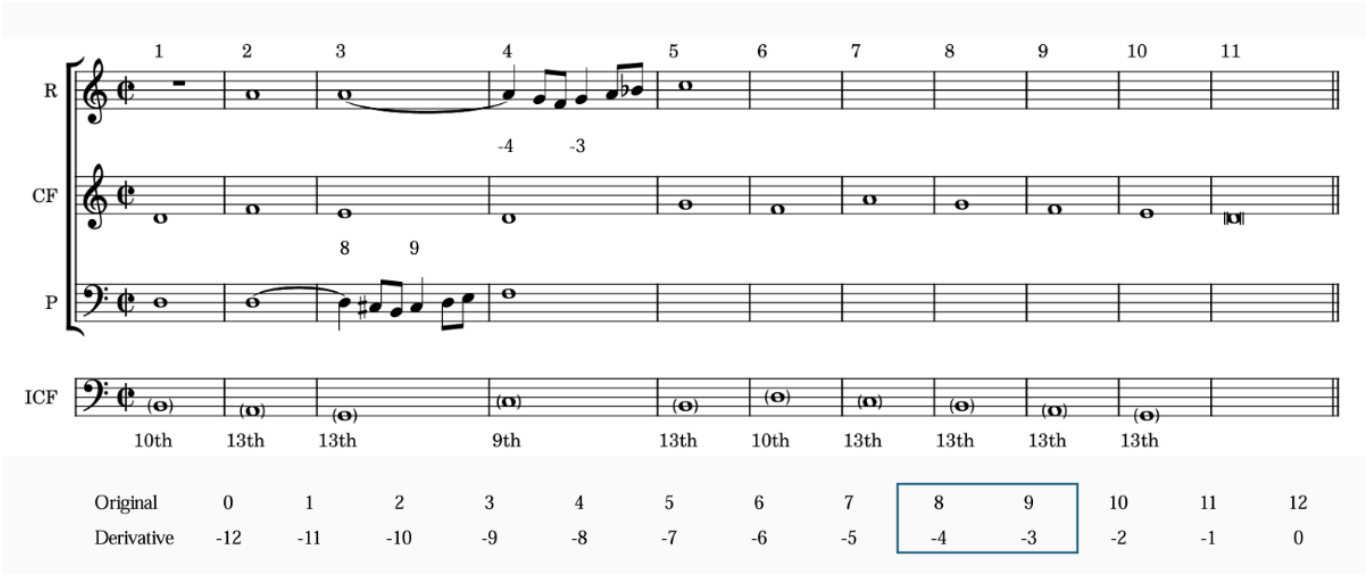
Original	0	1	2	3	4	5	6	7	8	9	10	11	12
Derivative	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0

Example 21. An option for the proposta in m. 4 using a fixed consonance



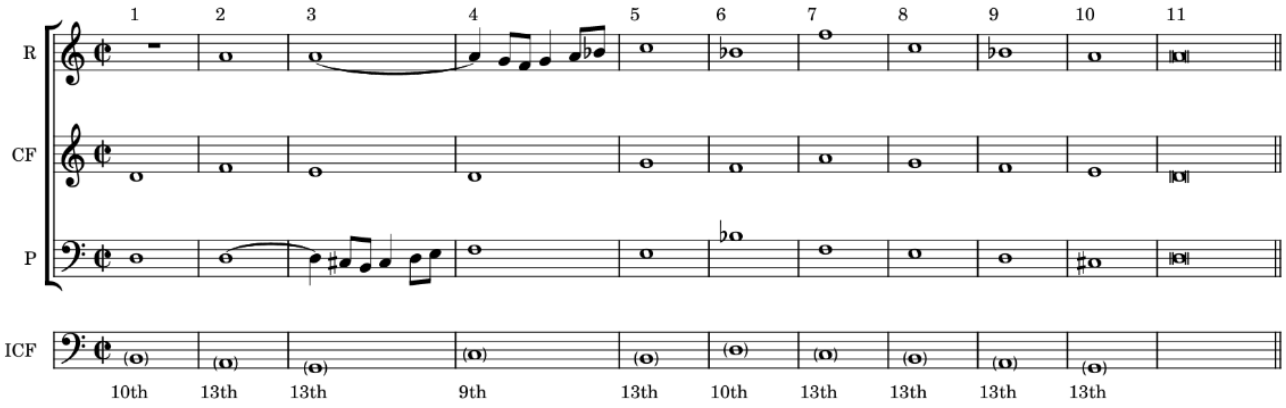
Original	0	1	2	3	4	5	6	7	8
Derivative	-8	-7	-6	-5	-4	-3	-2	-1	0

Example 22. An alternative for the proposta in m. 4 that uses variable intervals and exploits a change of function

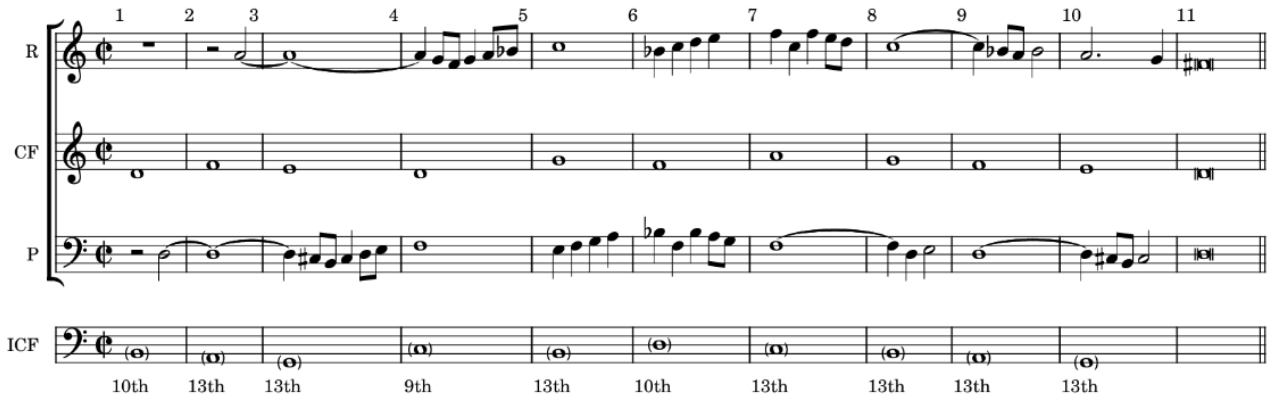


Original	0	1	2	3	4	5	6	7	8	9	10	11	12
Derivative	-12	-11	-10	-9	-8	-7	-6	-5	-4	-3	-2	-1	0

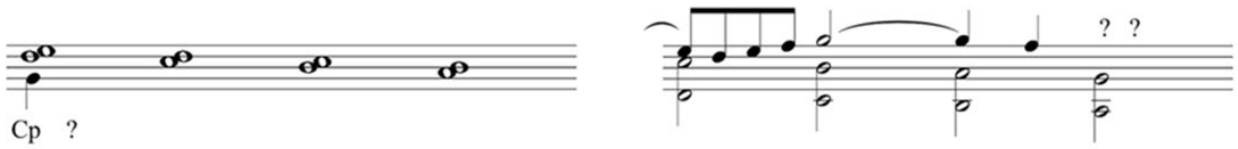
Example 23. Continuation and completion of the canonic framework



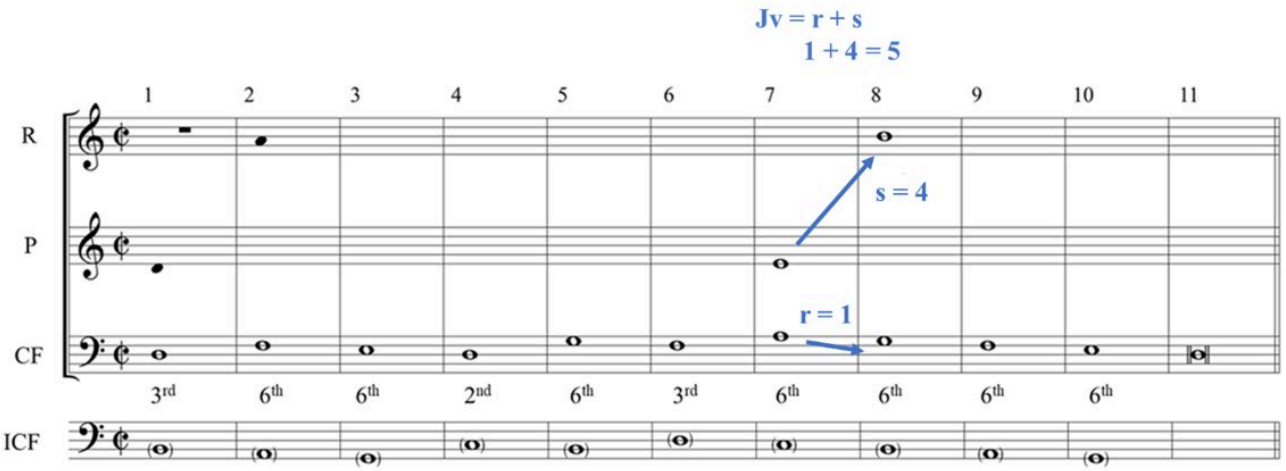
Example 24. Completing the canon with melodic and rhythmic fills



Example 25. Troublesome successions of guide intervals

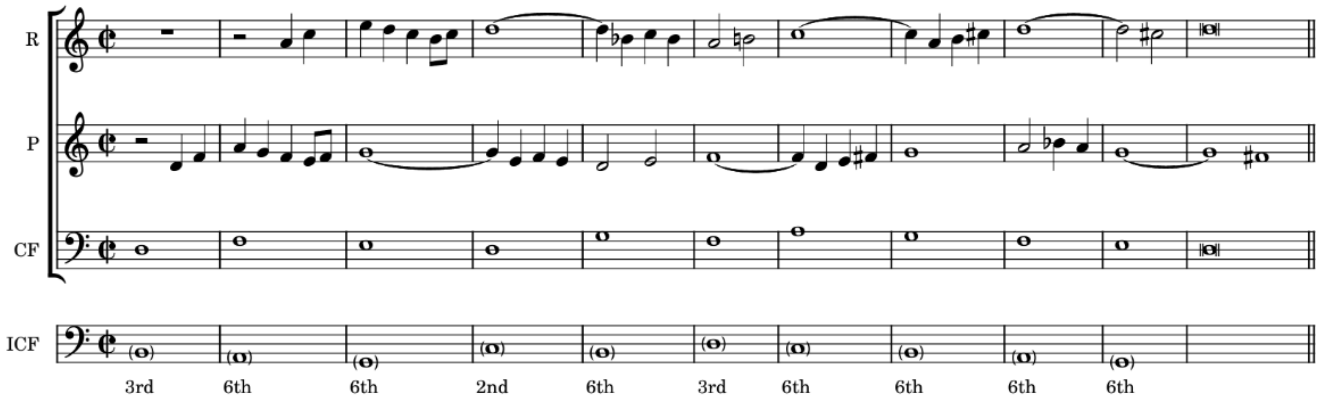


Example 26. An instance of a time unit governed by a direct vertical-shift index instead of an inverse shift

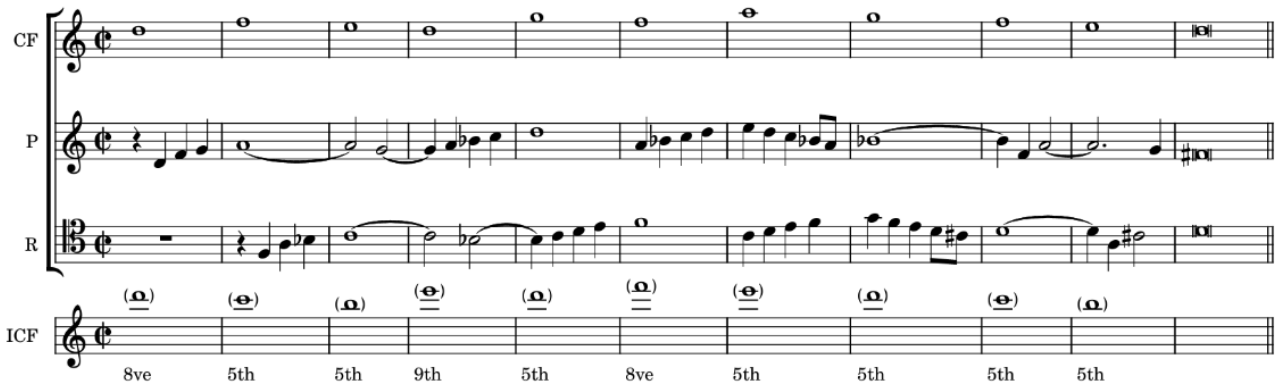


Original:	0	1	2	3	4	5	6	7	8	…
Derivative:	5	6	7	8	9	10	11	12	13	…
Original:	unison	2nd	3rd	4th	5th	6th	7th	8ve	9th	…
Derivative:	6th	7th	8ve	9th	10th	11th	12th	13th	14th	…

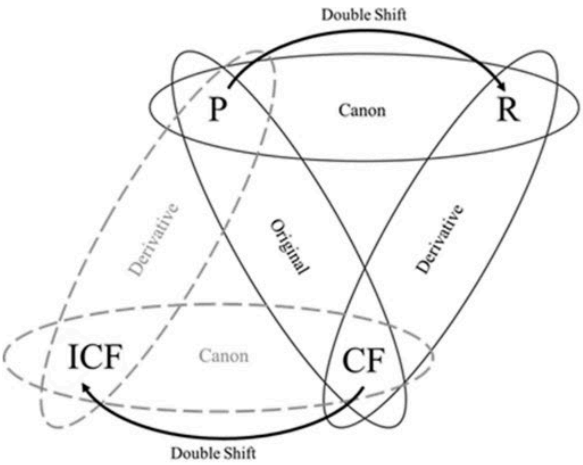
Example 27. A completed canon at the upper fifth at a distance of one whole note with the cantus firmus in the bass

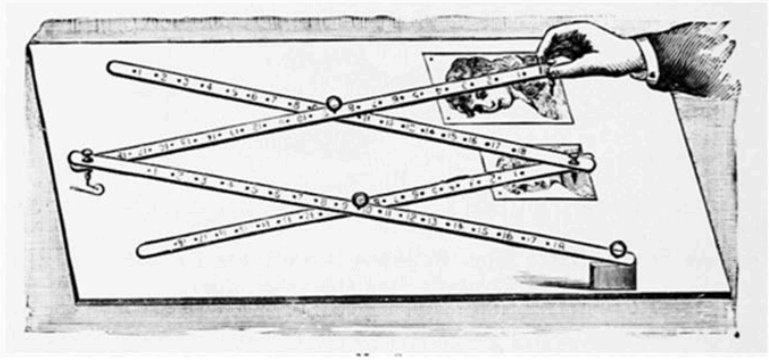


Example 28. A completed canon at the lower sixth at a distance of one whole note with the cantus firmus in the soprano

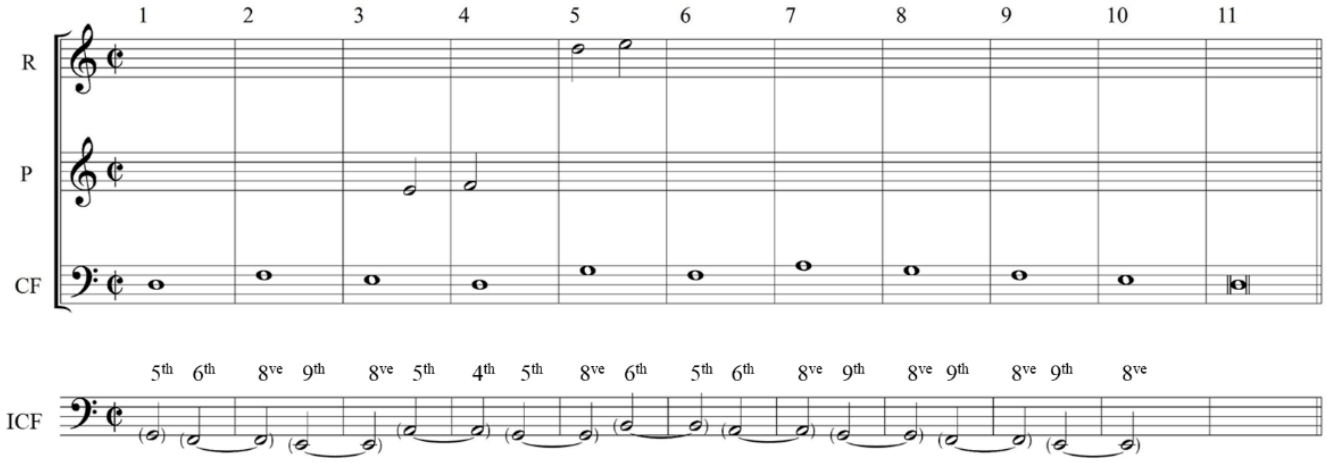


Example 29. A comparison of the diagram in Example 16 to a pantograph

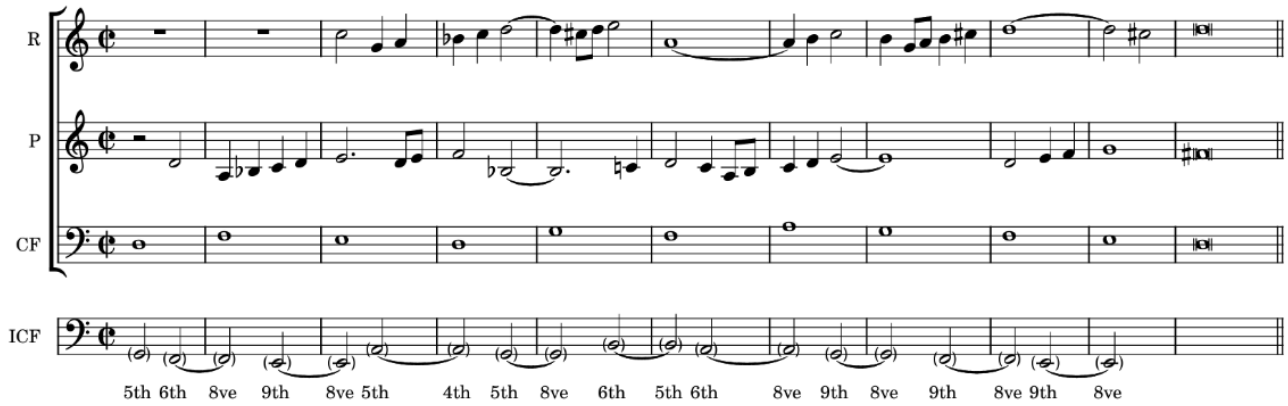




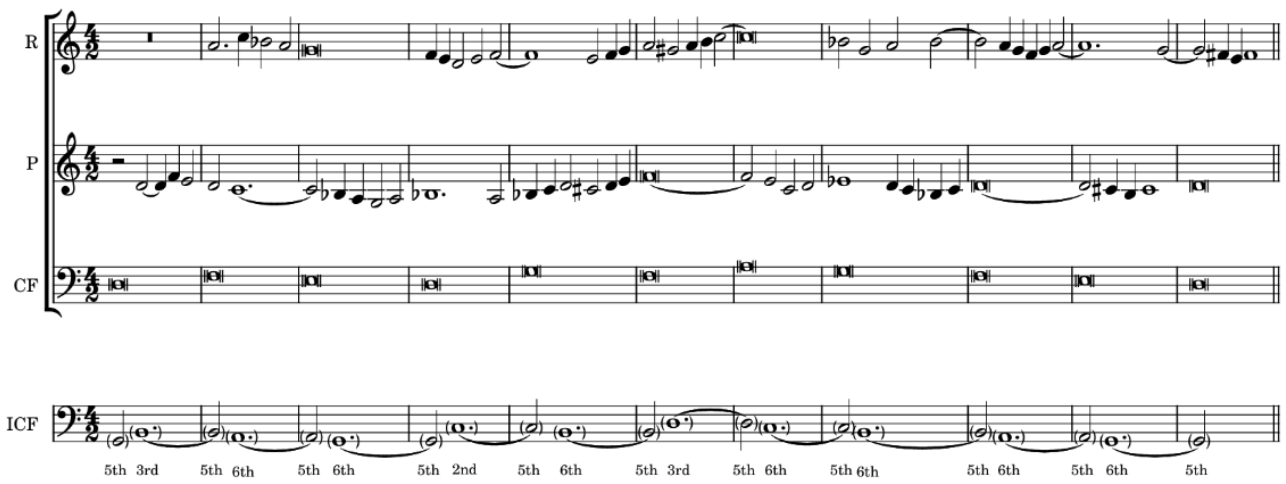
Example 30. Selecting a note for the proposta for a canon with a syncopated imaginary cantus firmus



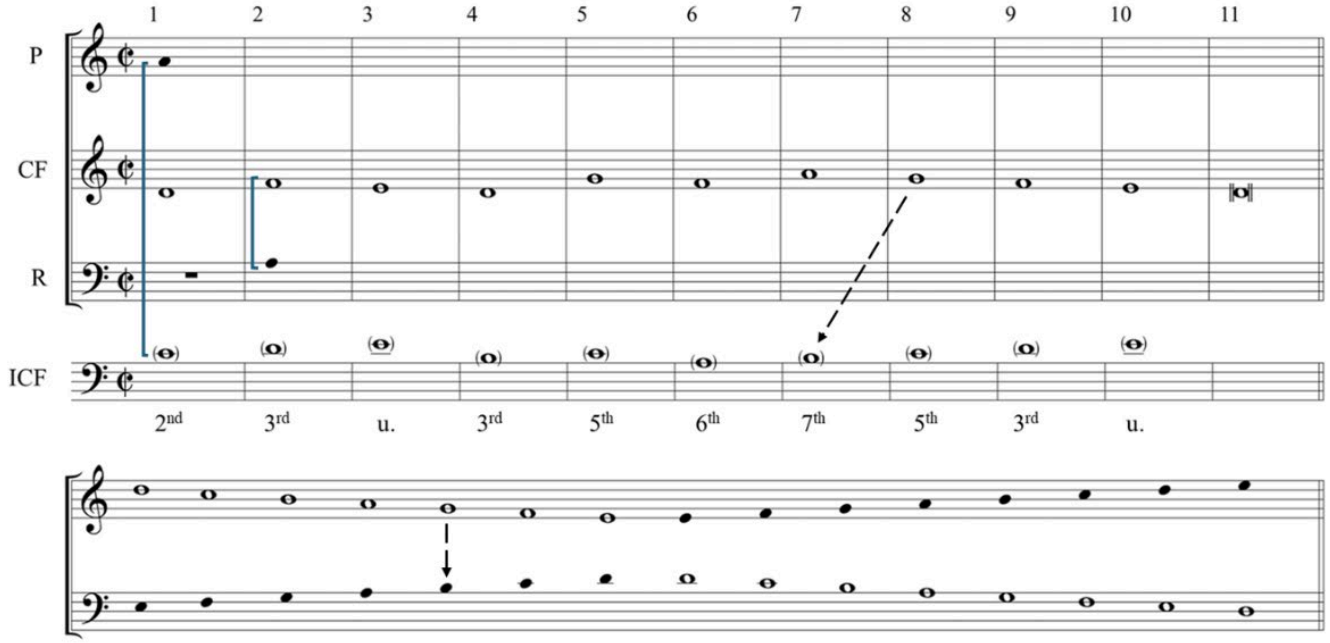
Example 31. A canon at the upper seventh at a distance of three half notes



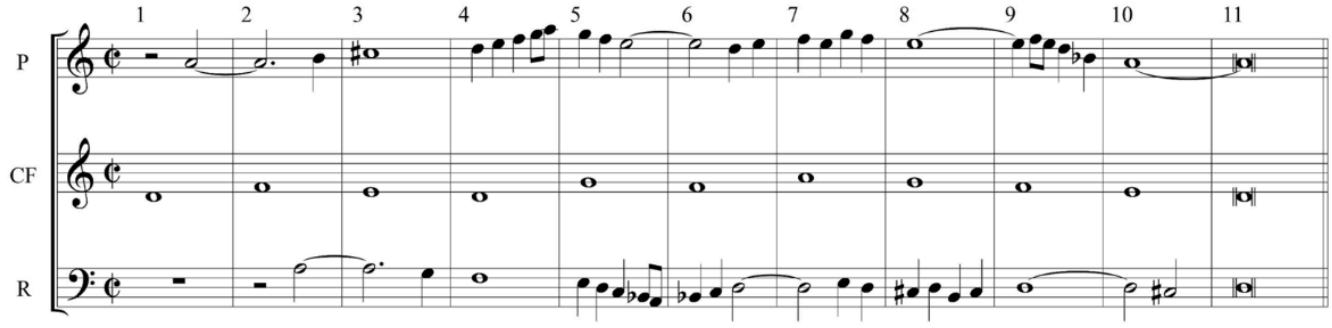
Example 32. A canon at the upper fifth at a distance of three half notes (*per arsin et thesin*)



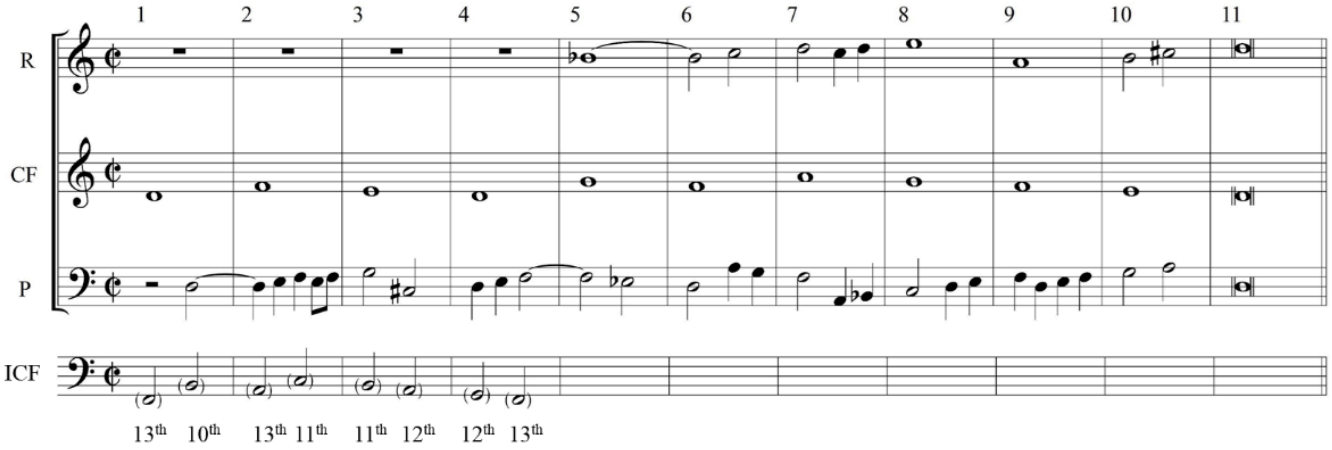
Example 33. The setup for a canon in contrary motion at the lower octave at a distance of one whole note, which must include the axis of inversion to find the placement of the imaginary cantus firmus



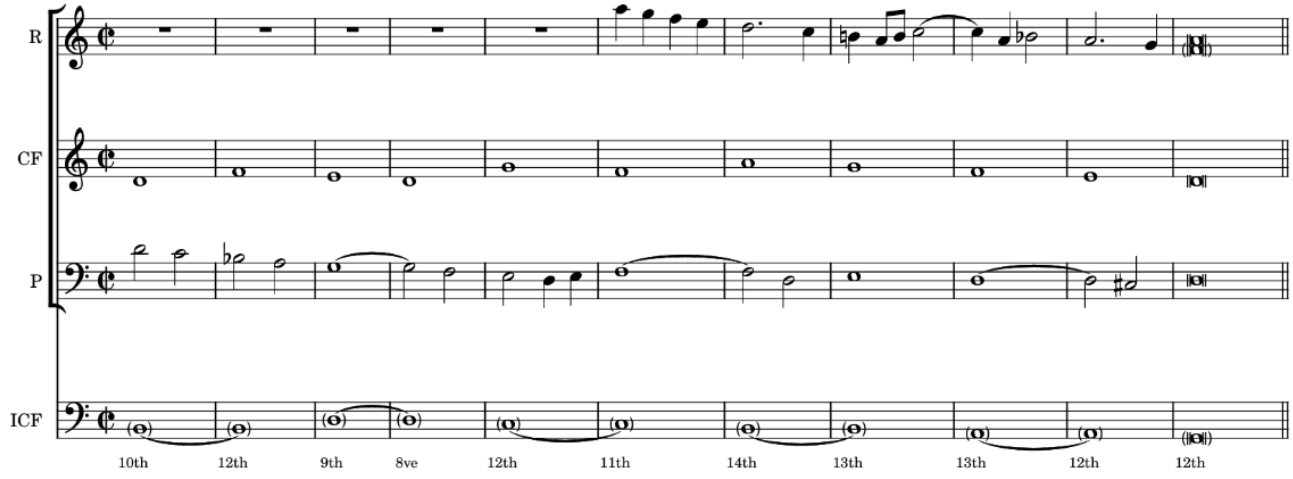
Example 34. A completed canon in contrary motion with a vertical axis of D4/E4 at a distance of one whole note



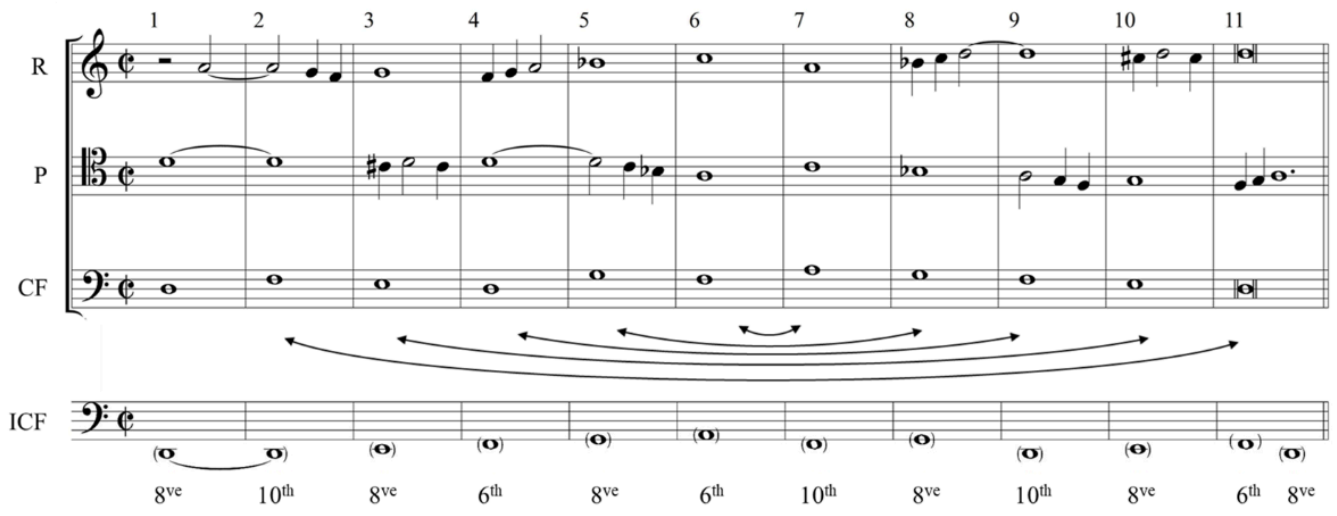
Example 35. A canon in augmentation at the upper 13th at a distance of three whole notes



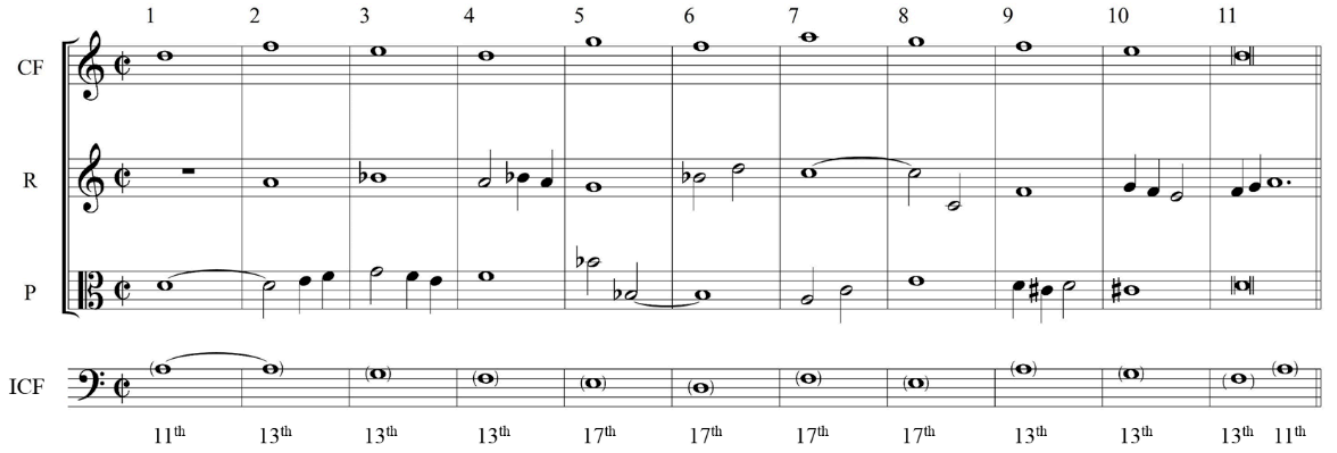
Example 36. A canon in diminution at the upper 12th at a distance of five whole notes



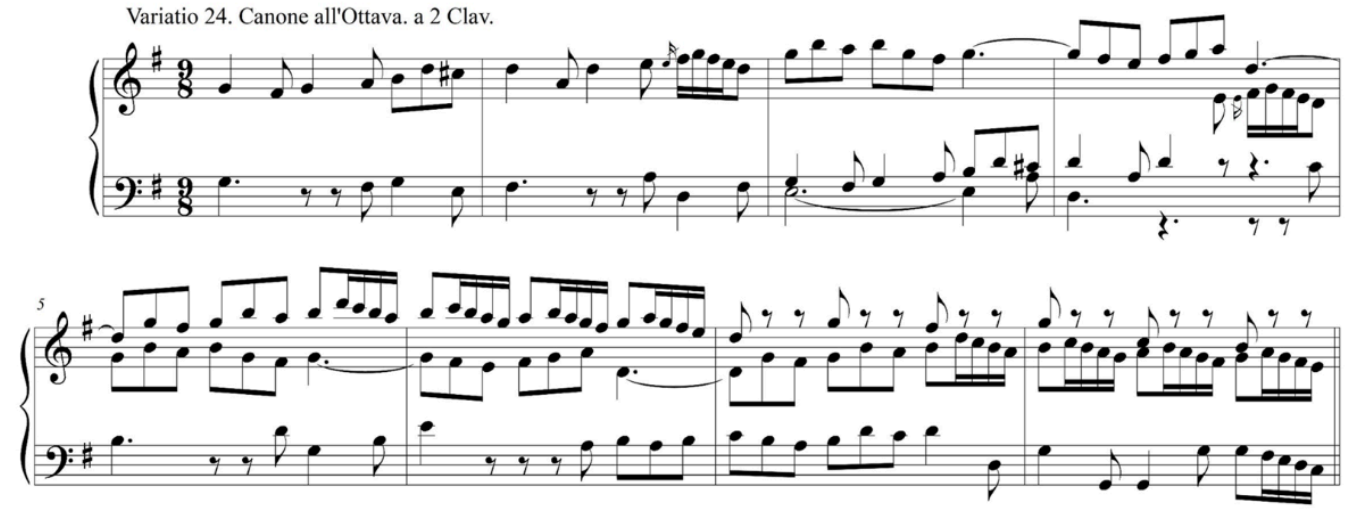
Example 37. A canon in retrograde motion at the upper octave with a horizontal axis between mm. 6 and 7



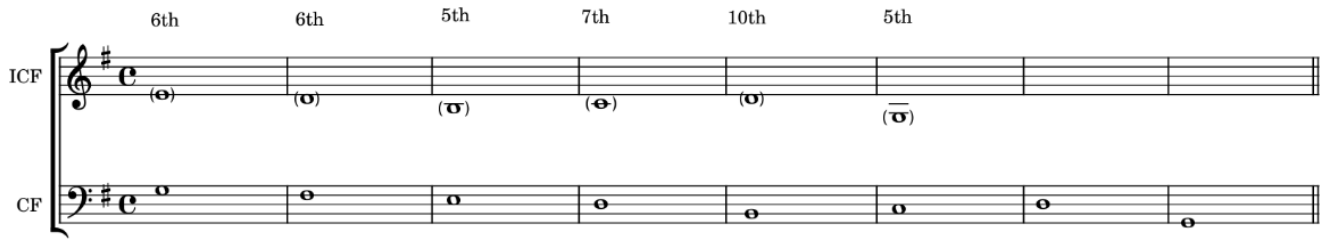
Example 38. A canon in inverted retrograde motion with a vertical axis at F4 and horizontal axis between mm. 6 and 7



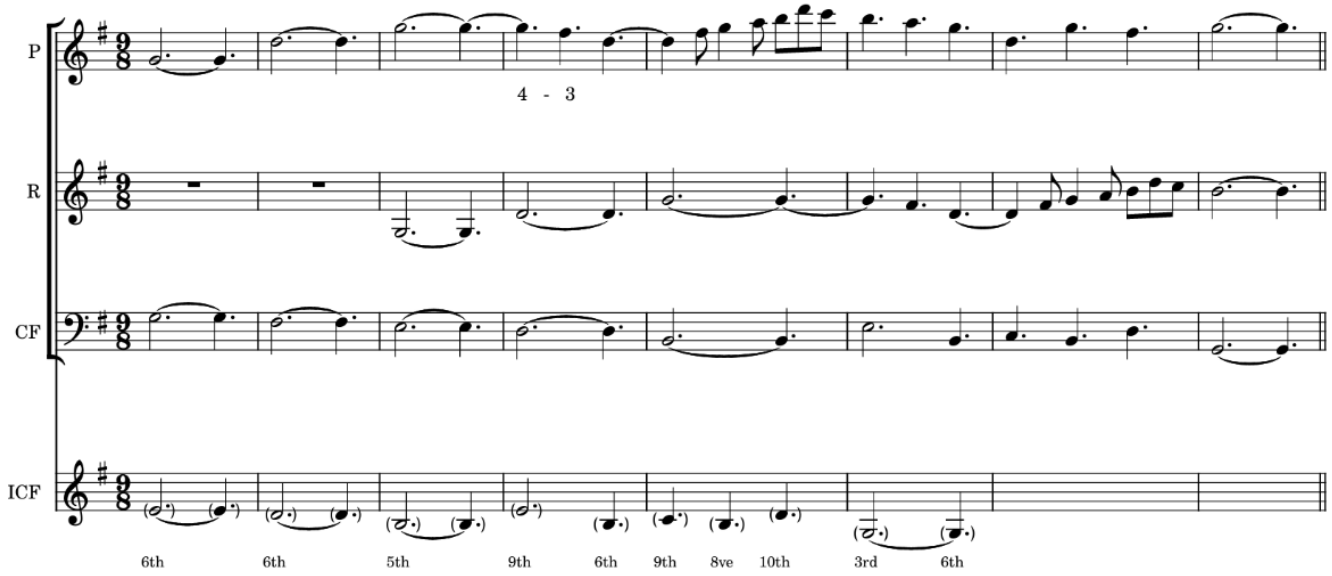
Example 39. J.S. Bach, Variation 24 from the *Goldberg Variations*, BWV 988, 1–8. (Kimiko Ishizaka, piano)



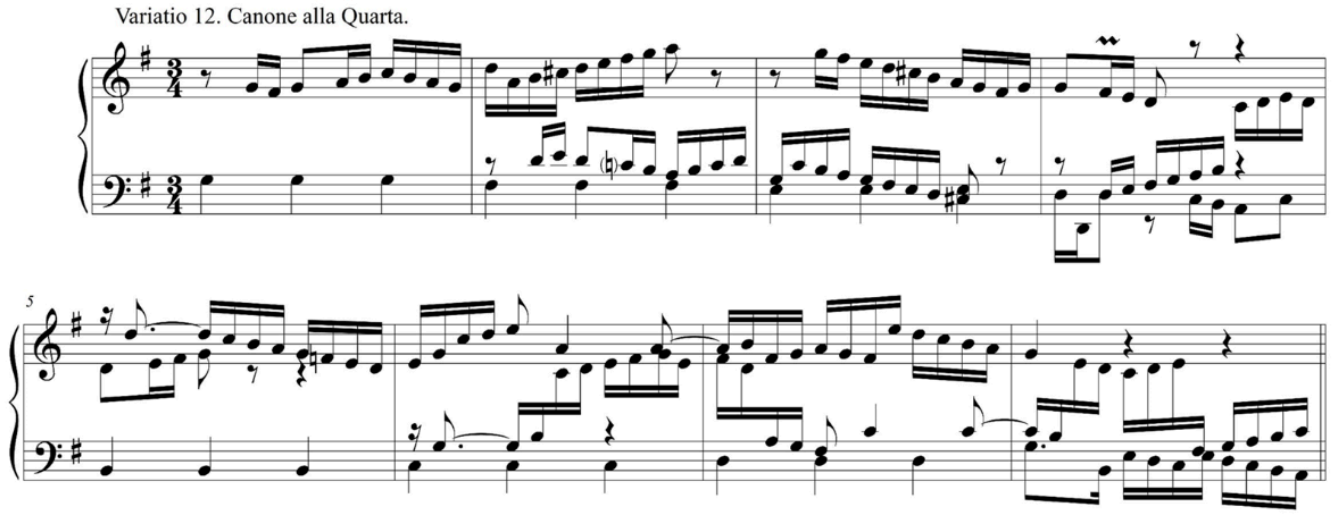
Example 40. The imaginary cantus firmus and the guide intervals for a canon at the lower octave at a distance of two whole notes assuming no changes to the first eight notes of the Goldberg bass line



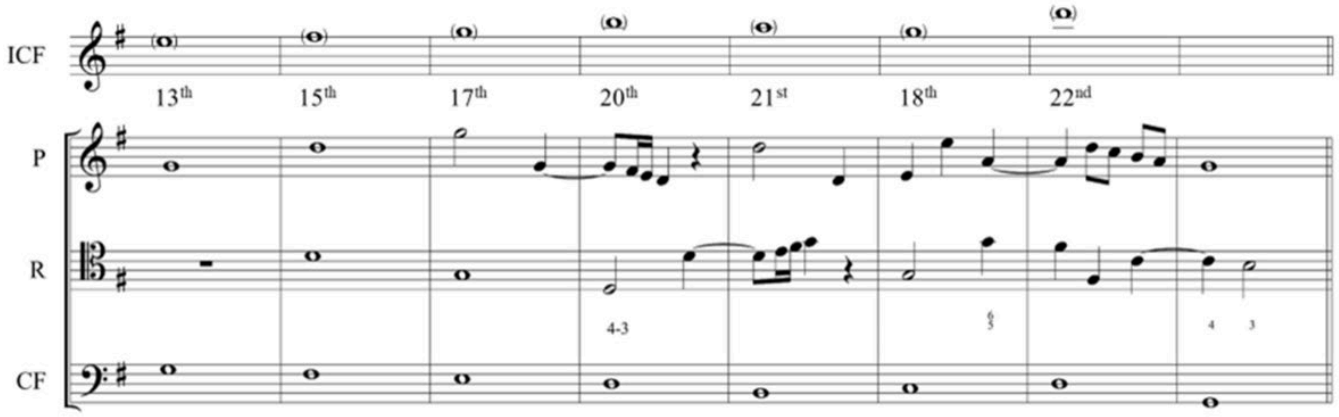
Example 41. An analysis of the essential voice leading of BWV 988 Variation 24, mm. 1–8 with imaginary cantus firmus and guide intervals indicated



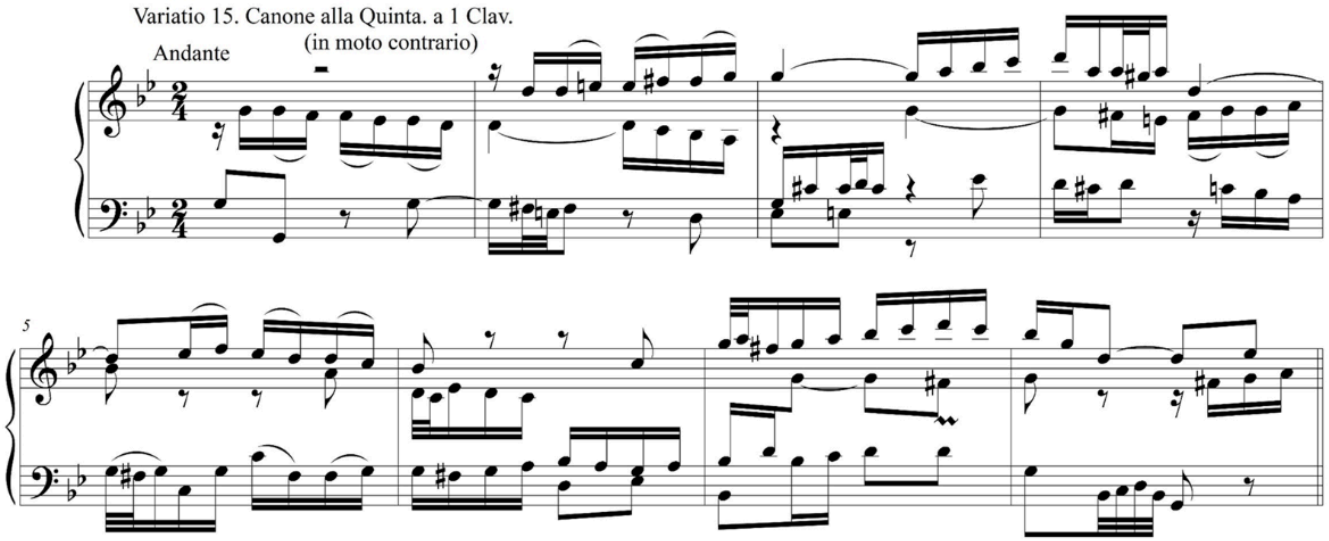
Example 42. J.S. Bach, Variation 12 from the *Goldberg Variations*, BWV 988, 1–8. (Kimiko Ishizaka, piano)



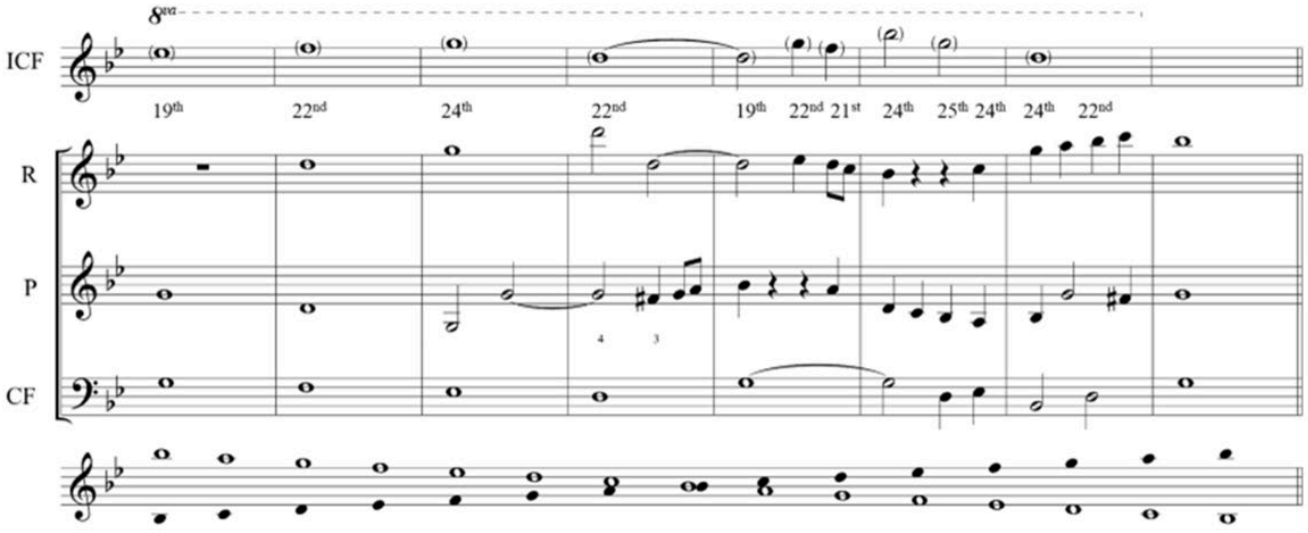
Example 43. An analysis of the essential voice leading of BWV 988, Variation 12, 1–8



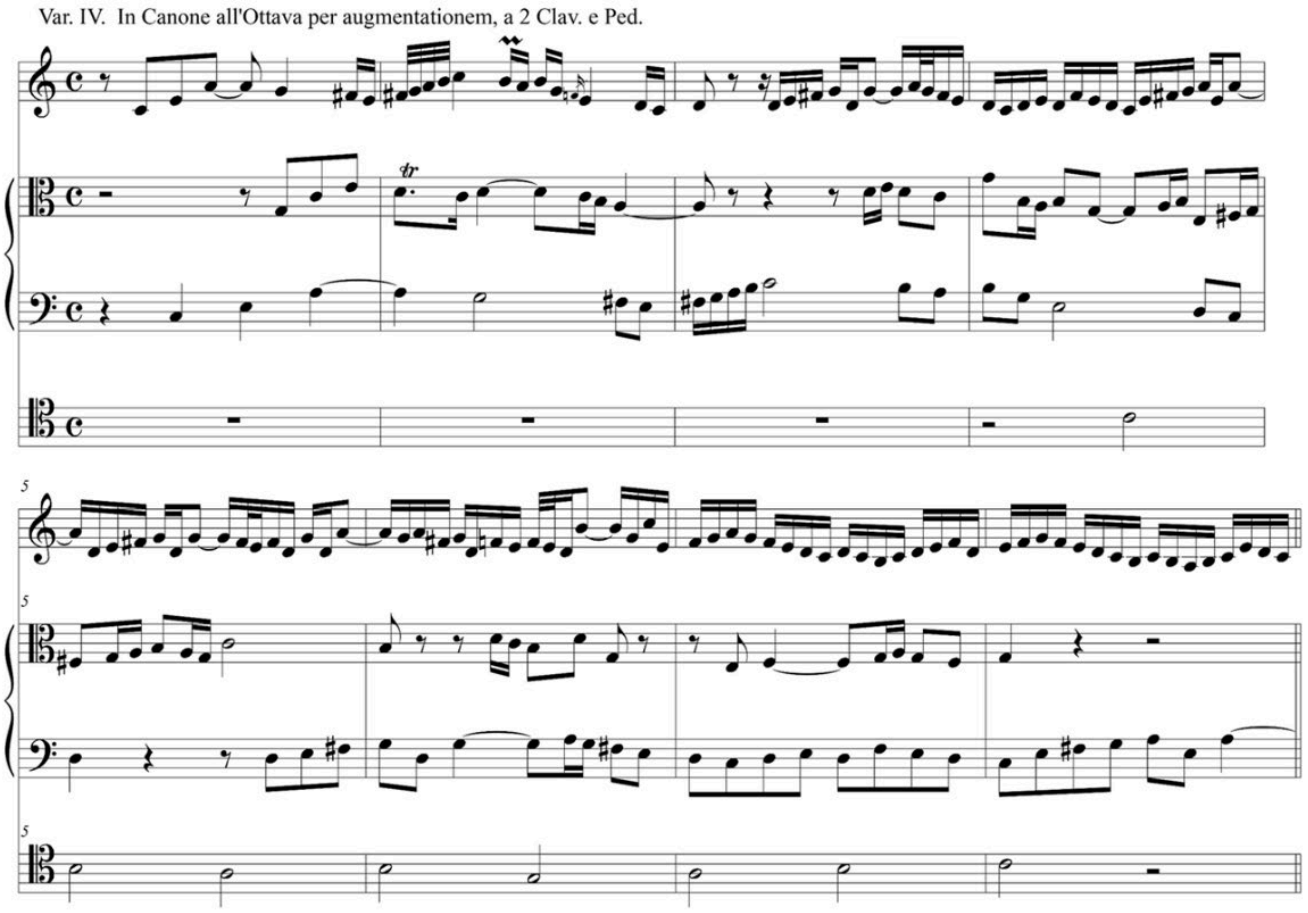
Example 44. J.S. Bach, Variation 15 from the *Goldberg Variations*, BWV 988, mm. 1–8. (Kimiko Ishizaka, piano)



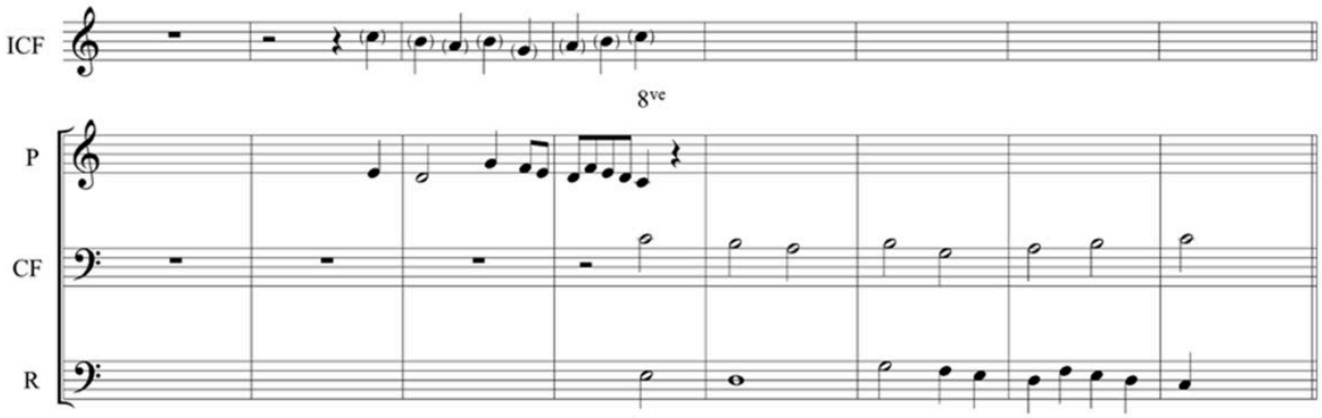
Example 45. An analysis of the essential voice leading of BWV 988, Variation 15, mm. 1–8



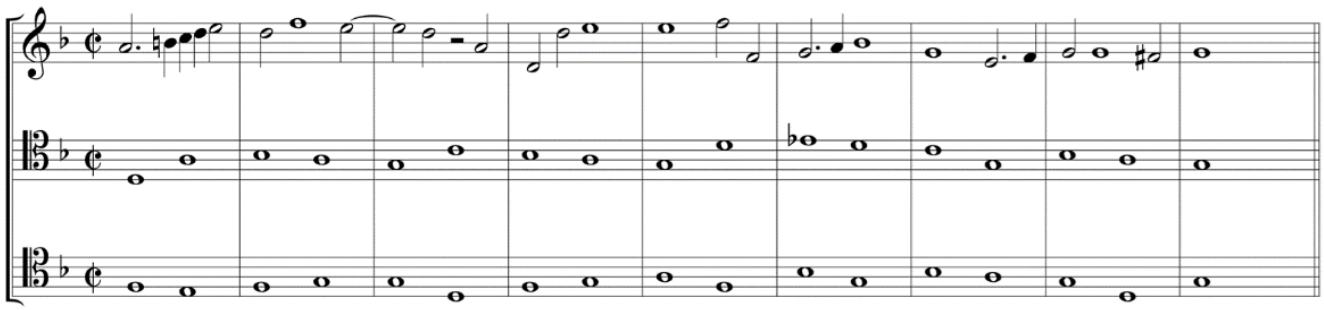
Example 46. J.S. Bach, Variation 4 from the *Canonic Variations on 'Vom Himmel hoch da komm' ich her,'* BWV 769, 1–8 (Carl Smith, organ)



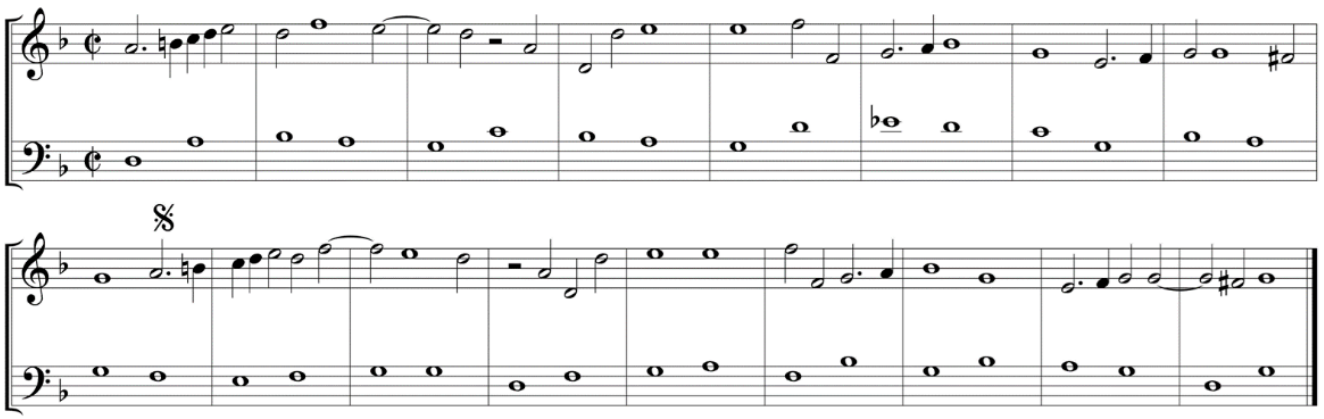
Example 47. An analysis of the essential voice leading of BWV 769, Variation 4, 1–8



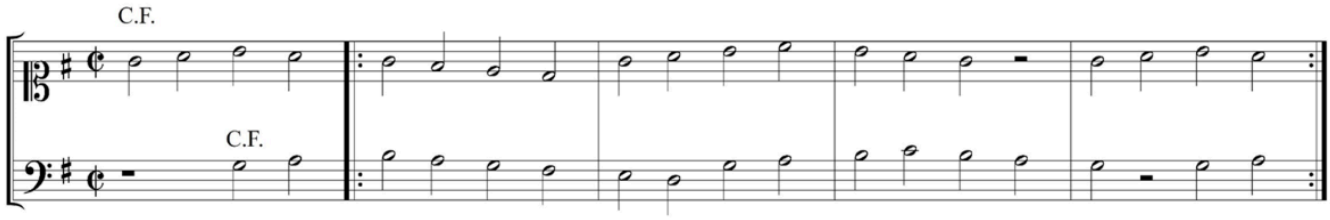
Example 48. A countermelody composed against two cantus firmi independently by Thomas Morley



Example 49. A realization of the repetition of Morley's countermelody as a melodic sequence in horizontal-shifting counterpoint



Example 50. A demonstration by Johann Anton André of how the chorale melody “Freu dich sehr o meine Seele” works in canon with itself at the lower octave



Example 51. André’s canon at the lower octave with the chorale melody as cantus firmus